Modeling switch reference in Koasati

Mary Moroney, *Cornell University*
mrm366@cornell.edu

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1 Introduction

- Switch reference (SR), a morphological phenomenon found in several languages in the world, is traditionally characterized as a way of indicating whether the subjects of two conjoined clauses are the same or different (Jacobsen 1993)
- Examples of SR in Koasati, a Muskogean language spoken in Louisiana and Texas, can be seen in (1)[1]

(1) Joekak roomkã itcokhalihkok
Joe-k room-˜ itcokhali:ka-**k**
Joe-SBJ room-OBJ enter-SS
'Joe came into the room,' (Rising 1992: 4)

a. Edkã hihcok cokko:lit
Ed-˜ hi:ca-**k** cokko:lit
Ed-OBJ see-SS sat_down
'saw Ed, and sat down.'

b. Edkã hihcan cokko:lit
Ed-˜ hi:ca-**n** cokko:lit
Ed-OBJ see-DS sat_down
'saw Ed, and he [Ed] sat down.'

- In (1), the morpheme *-k* (SS) in the first verb *itcokhalihkok* ('enter') indicates that its subject, *Joe*, is the same as the subject of the following verb, *hihcok*/*hihcan* ('see').
- In (1a), the *-k* (SS) on the second verb *hihcok* ('see') indicates that the subject of that verb, *Joe*, is the subject of the final verb *cokko:lit* ('sat down').
- In (1b), the *-n* (DS) on the second verb *hihcan* ('see') indicates that its subject, *Joe*, is not the subject of the final verb *cokko:lit* ('sat down'), but instead the object of *hihcan*, *Ed*, is.
- Consider the English equivalent of (1) in (2)

(2) Joe^j came into the room. He$_j$ saw Ed^k. He$_{j/k}$ sat down.

- *He* in the third sentence could refer to either Joe or Ed

[1] All data examples are copied unchanged from their sources except in the nasalization marker in examples from Kimball, which I changed from V̨ to Ṽ and in the third line of the gloss. The third line of the gloss has been changed to better fist the Leipzig glossing conventions.
Gloss abbreviations: SS = SAME SUBJECT; DS = DIFFERENT SUBJECT; SBJ = SUBJECT; OBJ = OBJECT

- The English is ambiguous where the Koasati is not
- Previous semantic analyses of SR include work by Stirling (1993) and McKenzie (2007, 2011, In review)
- They analyze SR as tracking events or situations, but I pursue a reference tracking analysis for Koasati SR
- I model this data on switch reference using Predicate Logic with Anaphora (PLA; Dekker 1994), a system that maintains an ordered list of individuals in a discourse

Roadmap

2 Koasati switch reference

- Koasati word order is typically SOV
- SR marking appears on the verb at the end of the clause
- The verbal SS and DS morphemes are homophonous with the nominal SBJ and OBJ markings

Morpheme	Attached to Noun	Attached to Verb
-k	subject (SBJ)	same subject (SS)
-n	object (OBJ)	different subject (DS)

Table 1: Subject, object, and switch reference morphemes

- The overlap in the form of the nominal subject and object marker with the SR markers suggests that there is an important connection between reference and SR

Notation for tables:

- Bold items in the table indicate overt arguments

(1a) Joekak roomkã itcokhalihkok Edkã hihcok cokko:lit
Joe-k room-˜ itcokhali:ka-**k** Ed-˜ hi:ca-**k** cokko:lit
Joe-SBJ room-OBJ enter-SS Ed-OBJ see-SS sat_down
'Joe came into the room, saw Ed, and sat down.' (Rising 1992: 4)

Clause	Verb Gloss	Subject	Object	SR Marker
1.	entered	**Joe**	**room**	SS
2.	see	Joe	**Ed**	SS
3.	sat_down	Joe	-	-

Table 2: Breakdown of (1a)

(1b) Joekak roomkã itcokhalihkok Edkã hihcan cokko:lit
Joe-k room-˜ itcokhali:ka-**k** Ed-˜ hi:ca-**n** cokko:lit
Joe-SBJ room-OBJ enter-SS Ed-OBJ see-DS sat_down
'Joe came into the room, saw Ed, and he [Ed] sat down.' (Rising 1992: 4)

Clause	Verb Gloss	Subject	Object	SR Marker
1.	enter	**Joe**	**room**	SS
2.	see	Joe	**Ed**	DS
3.	sat_down	Ed	-	-

Table 3: Breakdown of (1b)

- From these examples, it seems that there is a pattern to how individuals are introduced and referred back to
- Further, this pattern can be manipulated by the switch reference markers
  - The SS marker makes the subject and object of the SS marked clause the available subject and object, respectively, for the next clause
  - The DS marker makes the subject and object of the DS marked clause the available object and subject, respectively, for the next clause
- A system like PLA that can order individuals can be used to model this data

3 Background on PLA

- Predicate Logic with Anaphora (PLA; Dekker 1994) extends standard Predicate Logic in order to keep track of individuals in a discourse
- Has regular truth conditions, but a formula is interpreted as an update of an information state

(3) **A sample PLA information state**

$$s = \{\langle\ \underset{\mathsf{p}_2}{a},\ \underset{\mathsf{p}_1}{b},\ \underset{\mathsf{p}_0}{c}\ \rangle\}$$

- $\mathsf{p_i}$: i indexes the position of the pronoun
- $\exists$: introduces individuals to information state

(2) Joe$_j$ came into the room. He$_j$ saw Ed$_k$. He$_{j/k}$ sat down.

Table 4: Analysis of one translation of (2)

	English	PLA	Pro. Interpr.	Output State
a.				$s_0 = \{\langle\rangle\}$
b.	Joe$_j$ came into the room.	$\exists \mathsf{x}(\mathsf{x}=\mathsf{j} \wedge \exists \mathsf{y}(\mathsf{y}=\mathsf{r} \wedge \mathsf{Ixy}))$		$s_1 = \{\langle r, j\rangle\}$
c.	He$_j$ saw Ed$_k$.	$\exists \mathsf{y}(\mathsf{y}=\mathsf{e} \wedge \mathsf{Hp_0y})$	$[\mathsf{p_0}]_{s_1} = j$	$s_2 = \{\langle r, j, e\rangle\}$
d.	He$_k$ sat down.	$\mathsf{Cp_0}$	$[\mathsf{p_0}]_{s_2} = e$	$s_3 = \{\langle r, j, e\rangle\}$

- In (b), the narrow scope quantifier adds r to the information state first
- Then the broad scope quantifier adds j to the information state

Table 5: Analysis of other translation of (2)

	English	PLA	Pro. Interpr.	Output State
a.				$s_0 = \{\langle\rangle\}$
b.	Joe$_j$ came into the room.	$\exists \mathsf{x}(\mathsf{x}=\mathsf{j} \wedge \exists \mathsf{y}(\mathsf{y}=\mathsf{r} \wedge \mathsf{Ixy}))$		$s_1 = \{\langle r, j\rangle\}$
c.	He$_j$ saw Ed$_k$.	$\exists \mathsf{y}(\mathsf{y}=\mathsf{e} \wedge \mathsf{Hp_0y})$	$[\mathsf{p_0}]_{s_1} = j$	$s_2 = \{\langle r, j, e\rangle\}$
d.	He$_j$ sat down.	$\mathsf{Cp_1}$	$[\mathsf{p_1}]_{s_2} = j$	$s_3 = \{\langle r, j, e\rangle\}$

4 One list analysis

- In English the ambiguity of *he* is represented in PLA by different pronoun terms: $\mathsf{p_0}$ and $\mathsf{p_1}$
- The lack of ambiguity in the Koasati data can be captured by translating the subject agreement marker as $\mathsf{p_0}$ and object agreement marker as $\mathsf{p_1}$
- Further, the switch reference markers can be translated so that the DS marker swaps the order of the individuals in the $\mathsf{p_0}$ and $\mathsf{p_1}$ positions and the SS marker maintains the order

- a-SBJ: $\exists \mathsf{z}(\mathsf{z}=\mathsf{a})$
- b-OBJ: $\exists \mathsf{x}(\mathsf{x}=\mathsf{p_0} \wedge \exists \mathsf{z}(\mathsf{z}=\mathsf{b}))$
- intransitive verb: $\mathsf{Vp_0}$
- transitive verb: $\mathsf{Vp_0p_1}$
- SS: $\exists \mathsf{x}(\mathsf{x}=\mathsf{p_0} \wedge \exists \mathsf{y}(\mathsf{y}=\mathsf{p_1}))$
- DS: $\exists \mathsf{y}(\mathsf{y}=\mathsf{p_1} \wedge \exists \mathsf{x}(\mathsf{x}=\mathsf{p_0}))$

(4) **SS marker**

$$s_n = \{\langle\ a,\ b,\ c\ \rangle\} \xrightarrow{\text{SS}} s_{n+1} = \{\langle\ \langle a,\ b,\ c,\ b,\ c\ \rangle\}$$

(5) **DS marker**

$$s_n = \{\langle\ a,\ b,\ c\ \rangle\} \xrightarrow{\text{DS}} s_{n+1} = \{\langle\ \langle a,\ b,\ c,\ c,\ b\ \rangle\}$$

(1) Joekak roomkã itcokhalihkok
Joe-k room-˜ itcokhali:ka-**k**
Joe-SBJ room-OBJ enter-SS
'Joe came into the room,' (Rising 1992: 4)

Table 6: Analysis of (1)

	Gloss	PLA	Pronoun Interp.	Output State
a.	Joe-SBJ	$\exists \mathsf{z}(\mathsf{z}=\mathsf{j})$		$s_1 = \{\langle j\rangle\}$
b.	room-OBJ	$\exists \mathsf{x}(\mathsf{x}=\mathsf{p_0} \wedge \exists \mathsf{z}(\mathsf{z}=\mathsf{r}))$	$[\mathsf{p_0}]_{s_1} = j$	$s_2 = \{\langle j, r, j\rangle\}$
c.	enter	$\mathsf{Ip_0p_1}$	$[\mathsf{p_1}]_{s_2} = r, [\mathsf{p_0}]_{s_2} = j$	$s_3 = \{\langle j, r, j\rangle\}$
d.	-SS	$\exists \mathsf{x}(\mathsf{x}=\mathsf{p_0} \wedge \exists \mathsf{y}(\mathsf{y}=\mathsf{p_1}))$	$[\mathsf{p_1}]_{s_3} = r, [\mathsf{p_0}]_{s_3} = j$	$s_4 = \{\langle j, r, j, r, j\rangle\}$

(1a) Edkã hihcok cokko:lit
Ed-˜ hi:ca-**k** cokko:lit
Ed-OBJ see-SS sat_down
'saw Ed, and sat down.' (Rising 1992: 4)

Table 7: Analysis of (1a)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-OBJ	$\exists x(x = p_0 \wedge \exists z(z = e))$	$[p_0]_{s_4} = j$	$s_5 = \{\langle j,r,j,r,j,e,j\rangle\}$
f.	see	Hp_0p_1	$[p_1]_{s_5} = e, [p_0]_{s_5} = j$	$s_6 = \{\langle j,r,j,r,j,e,j\rangle\}$
g.	**-SS**	$\exists x(x = p_0 \wedge \exists y(y = p_1))$	$[p_1]_{s_6} = e, [p_0]_{s_6} = j$	$s_7 = \{\langle j,r,j,r,j,e,j,e,j\rangle\}$
h.	sat_down	Cp_0	$[p_0]_{s_7} = j$	$s_8 = \{\langle j,r,j,r,j,e,j,e,j\rangle\}$

(1b) Edkã hihcan cokko:lit
Ed-˜ hi:ca-**n** cokko:lit
Ed-OBJ see-DS sat_down
'saw Ed, and he [Ed] sat down.' (Rising 1992: 4)

Table 8: Analysis of (1b)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-OBJ	$\exists x(x = p_0 \wedge \exists z(z = e))$	$[p_0]_{s_4} = j$	$s_5 = \{\langle j,r,j,r,j,e,j\rangle\}$
f.	see	Hp_0p_1	$[p_1]_{s_5} = e, [p_0]_{s_5} = j$	$s_6 = \{\langle j,r,j,r,j,e,j\rangle\}$
g.	**-DS**	$\exists y(y = p_1 \wedge \exists x(x = p_0))$	$[p_1]_{s_6} = e, [p_0]_{s_6} = j$	$s_7 = \{\langle j,r,j,r,j,e,j,j,e\rangle\}$
h.	sat_down	Cp_0	$[p_0]_{s_7} = e$	$s_8 = \{\langle j,r,j,r,j,e,j,j,e\rangle\}$

- The different SR morpheme translations in (g) for **Tables 7-8** generate distinct unambiguous interpretations

5 A problem

- The data in (6) cannot be accounted for using the one list system

(6) Joekak roomkã itcokhali:kon
Joe-k room-˜ itcokhali:ka-**n**
Joe-SBJ room-OBJ enter-DS
'Joe came into the room,' (Rising 1992: 4)

a. Edkak hihcan cokko:lit
Ed-k hi:ca-**n** cokko:lit
Ed-SBJ see-DS sat_down
'Ed saw him, and Joe sat down.'

Clause	Verb Gloss	Subject	Object	SR Marker
1.	enter	**Joe**	**room**	DS
2.	see	**Ed**	Joe	DS
3.	sat_down	Joe	-	-

Table 9: Breakdown of (6)

Table 10: Analysis of (6)

	Gloss	PLA	Pronoun Interp.	Output State
a.	Joe-SBJ	$\exists z(z = j)$		$s_1 = \{\langle j\rangle\}$
b.	room-OBJ	$\exists x(x = p_0 \wedge \exists z(z = r))$	$[p_0]_{s_1} = j$	$s_2 = \{\langle j,r,j\rangle\}$
c.	enter	Ip_0p_1	$[p_1]_{s_2} = r, [p_0]_{s_2} = j$	$s_3 = \{\langle j,r,j\rangle\}$
d.	**-DS**	$\exists y(y = p_1 \wedge \exists x(x = p_0))$	$[p_1]_{s_3} = r, [p_0]_{s_3} = j$	$s_4 = \{\langle j,r,j,j,r\rangle\}$

Table 11: Analysis of (6a)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-SBJ	$\exists x(x = e)$		$s_5 = \{\langle j,r,j,j,\mathbf{r},e\rangle\}$
f.	see	Hp_0p_1	$[p_1]_{s_5} = \mathbf{r}, [p_0]_{s_5} = e$	$s_6 = \{\langle j,r,j,j,\mathbf{r},e,r\rangle\}$
g.	**-DS**	$\exists y(y = p_1 \wedge \exists x(x = p_0))$	$[p_1]_{s_6} = e, [p_0]_{s_6} = \mathbf{r}$	$s_7 = \{\langle j,r,j,j,r,e,e,\mathbf{r}\rangle\}$
h.	sat_down	Cp_0	$[p_0]_{s_7} = \mathbf{r}$	$s_8 = \{\langle j,r,j,j,r,e,e,\mathbf{r}\rangle\}$

5.1 Two list analysis

- I adapt PLA to be a two list system
- Bittner (2001) uses a two list system for anaphora and also applies it in an analysis of the obviative system in Kalallisut (West Greenlandic) (Bittner 2011)
- Little and Moroney (2016) use a two list system related to the one presented here in an analysis of obviation in Mi'gmaq

(7) **A sample two list information state**

$$\begin{array}{ccccccccc} s = \{\langle & \langle a, & b\rangle, & \langle c & d\rangle & \rangle\} \\ & \uparrow & \uparrow & \uparrow & \uparrow & \\ & p_1^{\top} & p_0^{\top} & p_1^{\perp} & p_0^{\perp} & \end{array}$$

- a-SBJ: $\exists^{\top}z(z = a)$
- b-OBJ: $\exists^{\perp}z(z = b)$
- intrans. verb: $Vp_0^{\top}$
- trans. verb: $Vp_0^{\top}p_0^{\perp}$
- SS: $\exists_{\perp}x(x = p_0^{\perp} \wedge \exists_{\perp}y(y = p_0^{\top})$
- DS: $\exists^{\top}y(y = p_0^{\perp}) \wedge \exists_{\perp}x(x = p_0^{\top})$

(8) **SS marker**

$$s_n = \{\langle \ \langle a, \ b\rangle, \ \langle c, \ d\rangle \ \rangle\} \xrightarrow{\text{SS}} s_{n+1} = \{\langle \ \langle a, \ b\rangle, \ \langle c, \ d, \ b, \ d\rangle \ \rangle\}$$

(9) **DS marker**

$$s_n = \{\langle \ \langle a, \ b\rangle, \ \langle c, \ d\rangle \ \rangle\} \xrightarrow{\text{DS}} s_{n+1} = \{\langle \ \langle a, \ b, \ d\rangle, \ \langle c, \ d, \ b\rangle \ \rangle\}$$

5.2 Accounting for data

- The two list system can still account for the initial data:

Table 12: Analysis of (1)

	Gloss	PLA	Pronoun Interp.	Output State
a.	Joe-SBJ	$\exists^\top z(z=j)$		$s_1=\{\langle\langle j\rangle,\langle\rangle\rangle\}$
b.	room-OBJ	$\exists^\perp z(z=r)$		$s_2=\{\langle\langle j\rangle,\langle r\rangle\rangle\}$
c.	enter	$I p_0^\top p_0^\perp$	$[p_0^\top]_{s_2}=j,[p_0^\perp]_{s_2}=r$	$s_3=\{\langle\langle j\rangle,\langle r\rangle\rangle\}$
d.	**-SS**	$\exists^\perp x(x=p_0^\perp\wedge\exists^\perp y(y=p_0^\top))$	$[p_0^\perp]_{s_3}=r,[p_0^\top]_{s_3}=j$	$s_4=\{\langle\langle j\rangle,\langle r,j,r\rangle\rangle\}$

Table 13: Analysis of (1a)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-OBJ	$\exists^\perp z(z=e)$		$s_5=\{\langle\langle j\rangle,\langle r,j,r,e\rangle\rangle\}$
f.	see	$H p_0^\top p_0^\perp$	$[p_0^\top]_{s_5}=j,[p_0^\perp]_{s_5}=e$	$s_6=\{\langle\langle j\rangle,\langle r,j,r,e\rangle\rangle\}$
g.	**-SS**	$\exists^\perp x(x=p_0^\perp\wedge\exists^\perp y(y=p_0^\top))$	$[p_0^\top]_{s_6}=j,[p_0^\perp]_{s_6}=e$	$s_7=\{\langle\langle j\rangle,\langle r,j,r,e,j,e\rangle\rangle\}$
h.	sat_down	$C p_0^\top$	$[p_0^\top]_{s_7}=j$	$s_8=\{\langle\langle j\rangle,\langle r,j,r,e,j,e\rangle\rangle\}$

Table 14: Analysis of (1b)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-OBJ	$\exists^\perp z(z=e)$		$s_5=\{\langle\langle j\rangle,\langle r,j,r,e\rangle\rangle\}$
f.	see	$H p_0^\top p_0^\perp$	$[p_0^\top]_{s_5}=j,[p_0^\perp]_{s_5}=e$	$s_6=\{\langle\langle j\rangle,\langle r,j,r,e\rangle\rangle\}$
g.	**-DS**	$\exists y(y=p_0^\perp)\wedge\exists^\perp x(x=p_0^\top)$	$[p_0^\top]_{s_6}=j,[p_0^\perp]_{s_6}=e$	$s_7=\{\langle\langle j,e\rangle,\langle r,j,r,e,j\rangle\rangle\}$
h.	sat_down	$C p_0^\top$	$[p_0^\top]_{s_7}=e$	$s_8=\{\langle\langle j,e\rangle,\langle r,j,r,e,j\rangle\rangle\}$

- It can account for the problematic data by keeping the available subject and object individuals separate:

Table 15: Analysis of (6)

	Gloss	PLA	Pronoun Interp.	Output State
a.	Joe-SBJ	$\exists z(z=j)$		$s_1=\{\langle\langle j\rangle,\langle\rangle\rangle\}$
b.	room-OBJ	$\exists^\perp z(z=r)$		$s_2=\{\langle\langle j\rangle,\langle r\rangle\rangle\}$
c.	enter	$I p_0^\top p_0^\perp$	$[p_0^\top]_{s_2}=j,[p_0^\perp]_{s_2}=r$	$s_3=\{\langle\langle j\rangle,\langle r\rangle\rangle\}$
d.	**-DS**	$\exists y(y=p_0^\perp)\wedge\exists^\perp x(x=p_0^\top)$	$[p_0^\perp]_{s_3}=r,[p_0^\top]_{s_3}=j$	$s_4=\{\langle\langle j,r\rangle,\langle r,j\rangle\rangle\}$

Table 16: Analysis of (6a)

	Gloss	PLA	Pronoun Interp.	Output State
e.	Ed-SBJ	$\exists z(z=e)$		$s_5=\{\langle\langle j,r,e\rangle,\langle r,j\rangle\rangle\}$
f.	see	$H p_0^\top p_0^\perp$	$[p_0^\top]_{s_5}=e,[p_0^\perp]_{s_5}=j$	$s_6=\{\langle\langle j,r,e\rangle,\langle r,j\rangle\rangle\}$
g.	**-DS**	$\exists y(y=p_0^\perp)\wedge\exists^\perp x(x=p_0^\top)$	$[p_0^\top]_{s_6}=e,[p_0^\perp]_{s_6}=j$	$s_7=\{\langle\langle j,r,e,j\rangle,\langle r,j,e\rangle\rangle\}$
h.	sat_down	$C p_0^\top$	$[p_0^\top]_{s_7}=j$	$s_8=\{\langle\langle j,r,e,j\rangle,\langle r,j,e\rangle\rangle\}$

6 Conclusion

- I have presented basic data of switch reference in Koasati
- I have discussed two PLA analyses for how to account for this data
  - One account uses Dekker's (1994) one-list system
  - The other account modifies his system to two lists to separate subjects and objects
- The two list analysis is better equipped to capture the data
- There is more work to be done to capture more complex data (plurals, indexicals, ditransitives)

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A Two list fragment

- Additions to PLA are indicated with a $*$

DEFINITION 1.1 (Basic Expressions of PLA)

1. $C = \{\mathsf{a}, \mathsf{b}, \dots \mathsf{n}\}$ (entity) constants
2. $V = \{\mathsf{x}, \mathsf{y}, \mathsf{z}, \mathsf{x}', \mathsf{y}', \mathsf{z}', \dots\}$ (entity) variables

$*$3. $A = \{\mathsf{p}_\mathsf{i}^\top \mid \mathsf{i} \in \mathscr{N}\}$ (entity) pronouns of list $\top$

$*$4. $B = \{\mathsf{p}_\mathsf{i}^\perp \mid \mathsf{i} \in \mathscr{N}\}$ (entity) pronouns of list $\perp$

$*$5. $T = C \cup V \cup A \cup B$ (entity) terms

6. $R^n = \{\mathsf{A}^1, \dots, \mathsf{A}^n, \mathsf{B}^1, \dots, \mathsf{Z}^n\}$ n-ary predicates

DEFINITION 1.2 (Syntax of PLA) The set L of PLA formulas is the smallest set such that:

1. if $\mathsf{t}_1, \dots, \mathsf{t}_\mathsf{n} \in T$ and $\mathsf{R} \in R^n$, then $\mathsf{Rt}_1 \dots \mathsf{t}_\mathsf{n} \in L$
2. if $\mathsf{t}_1, \mathsf{t}_2 \in T$, then $\mathsf{t}_1 = \mathsf{t}_2 \in L$
3. if $\phi \in L$, then $\neg\phi \in L$

$*$4. if $\phi \in L$ and $x \in V$, then $\exists^\top \mathsf{x}\phi \in L$

$*$5. if $\phi \in L$ and $x \in V$, then $\exists^\perp \mathsf{x}\phi \in L$

6. if $\phi, \psi \in L$, then $(\phi \wedge \psi) \in L$

DEFINITION 2.1 (Information States)

$*$1. $S^n = \mathscr{P}(D^a \times D^b)$ the set of information states about n subjects, where a is the number of subject in the $\top$ list and b is the number of subjects in the $\perp$ list and $a + b = n$

2. $S = \cup_{n \in \mathscr{N}} S^n$ the set of information states

$*$3. For a state $s \in S^n$, where $a + b = n$ and $0 < j \leq a$, and for any case $e = \langle\langle d_1^\top, \dots, d_a^\top\rangle, \langle d_1^\perp, \dots, d_b^\perp\rangle\rangle \in s$, $d_j^\top$ is a possible value for the j-th subject of s, also indicated as $e_j^\top$.

$*$4. For a state $s \in S^n$, where $a + b = n$ and $0 < k \leq b$, and for any case $e = \langle\langle d_1^\top, \dots, d_a^\top\rangle, \langle d_1^\perp, \dots, d_b^\perp\rangle\rangle \in s$, $d_k^\perp$ is a possible value for the k-th subject of s, also indicated as $e_k^\perp$.

$*$5. $s_0 = \{\langle\langle\rangle, \langle\rangle\rangle\}$ (the initial state of information: $D^0 \times D^0$)

$*$6. $\top^n = D^a \times D^b$ (the minimal state of information about n subjects, where $a + b = n$)

$*$7. $\{e\}$ for any $e = \langle\langle d_1^\top, \dots, d_a^\top\rangle, \langle d_1^\perp, \dots, d_b^\perp\rangle\rangle \in D^a \times D^b$ (the maximal state of information about n subjects, where $a + b = n$)

8. $\perp^n = \{\}$ (the absurd information state about n subjects, where $n > 0$)

DEFINITION 2.2 (Notational Convention)

1. If $e \in D^n$ and $e' \in D^m$, then $e \cdot e' = \langle e_1, \dots e_n, e'_1, \dots, e'_m\rangle \in D^{n+m}$
2. e' is an extension of e, $e \leq e'$, iff $\exists e'' : e' = e \cdot e''$

$*$3. e' is an extension of e, $e \leq e'$, iff $\forall e^{\top\prime} \in e'\ \exists e^\top \in e : e^\top \leq e^{\top\prime}$ & $\forall e^{\perp\prime} \in e' \exists e^\perp \in e : e^\perp \leq e^{\perp\prime}$

$*$4. For $s \in S^n (i \in D^n)$, $N_s = n (= a + b)$, $N_a = a$, $N_b = b$, the number of subjects of $s(i)$

DEFINITION 2.3 (Information Update)

1. State s' is an update of state $s, s \leq s'$, iff $N_s \leq N_{s'}$, and $\forall e' \in s' \exists e \in s : e \leq e'$

DEFINITION 3.1 (Interpretation of Terms)

1. $[\mathsf{c}]_{\mathscr{M},s,e,g} = F(\mathsf{c})$ for all constants c
2. $[\mathsf{x}]_{\mathscr{M},s,e,g} = g(\mathsf{x})$ for all variables x

$*$3. $[p_i^\top]_{\mathscr{M},s,e,g} = e^\top_{N_\top - i}$ for all pronouns $p_i^\top$ and e and $e^\top$ and s such that $e^\top \in e$ and $e \in s$ and $N_s > i$

$*$4. $[p_i^\perp]_{\mathscr{M},s,e,g} = e^\perp_{N_k - i}$ for all pronouns $p_i^\perp$ and e and $e^\perp$ and s such that $e^\perp \in e$ and $e \in s$ and $N_s > i$

DEFINITION 3.2 (Semantics of PLA)

1. $s[\![\mathsf{Rt}_1 \dots \mathsf{t}_\mathsf{n}]\!]_{\mathscr{M},g} = \{e \in s \mid \langle[\mathsf{t}_1]_{\mathscr{M},s,e,g}, \dots, [\mathsf{t}_\mathsf{n}]_{\mathscr{M},s,e,g}\rangle \in F(\mathsf{R})\}$ (if $N_s > I_{\mathsf{t}_1,\dots,\mathsf{t}_\mathsf{n}}$)
2. $s[\![\mathsf{t}_1 = \mathsf{t}_2]\!]_{\mathscr{M},g} = \{e \in s \mid [\mathsf{t}_1]_{\mathscr{M},s,e,g} = [\mathsf{t}_2]_{\mathscr{M},s,e,g}\}$
3. $s[\![\neg\phi]\!]_{\mathscr{M},g} = \{e \in s \mid \neg\exists e' : e \leq e'$ & $e' \in s[\![\phi]\!]_{\mathscr{M},g}\}$

$*$4. $s[\![\exists^\top x\phi]\!]_{\mathscr{M},g} = \{\langle e^\top \cdot d, e^\perp\rangle \mid d \in D$ & $\langle e^\top, e^\perp\rangle \in s[\![\phi]\!]_{\mathscr{M},g[x/d]}\}$

$*$5. $s[\![\exists^\perp x\phi]\!]_{\mathscr{M},g} = \{\langle e^\top, e^\perp \cdot d\rangle \mid d \in D$ & $\langle e^\top, e^\perp\rangle \in s[\![\phi]\!]_{\mathscr{M},g[x/d]}\}$

6. $s[\![\phi \wedge \psi]\!]_{\mathscr{M},g} = s[\![\phi]\!]_{\mathscr{M},g}[\![\psi]\!]_{\mathscr{M},g}$

DEFINITION 4.1 (Support and Entailment)

1. s supports ϕ wrt $\mathscr{M}$ and g, $s \vDash_{\mathscr{M},g} \phi$ iff $\forall e \in s : \exists e' : e \leq e'$ & $e' \in s[\![\phi]\!]_{\mathscr{M},g}$
2. $\phi_1, \dots, \phi_n$ entail $\psi, \phi_1, \dots, \phi_n \vDash \psi$ iff $\forall \mathscr{M}, g\ \forall s \in S : s[\![\phi_1]\!]_{\mathscr{M},g} \dots [\![\phi_n]\!]_{\mathscr{M},g} \vDash_{\mathscr{M},g} \psi$ (if defined)