The Geometry of Pictorial Representation

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This essay develops the thesis that pictorial representation is grounded in the geometrical transformation of projection from a viewpoint. This idea is articulated as a semantic principle, here termed the Projection Principle. Understanding pictorial contents as viewpoint-centered spaces, the principle holds, roughly, that for a picture to express a space as content, the picture itself must be a geometrical projection of that space. As a corollary, for a picture to accurately depict a given scene, it must be possible to derive the picture by geometrical projection from that scene. I’ll argue that the Projection Principle provides the spatial and chromatic foundation for any further elaboration of pictorial content, and describes characteristic structure of all forms of pictorial representation.

The idea that projection is in some way essentially tied to pictorial representation is hundreds of years old at least. Recently, Howell (1974), Hyman (2006), Kulvicki (2006), and Greenberg (2013) have made the specific suggestion that projection imposes a direct constraint on pictorial content. This essay aims to improve on these accounts, in the first place by giving the core idea a new and more precise expression, and by clarifying the key motivations behind it. Further, whereas previous work has often focussed on an artificially narrow band of pictorial styles, or systems of depiction, the version of the Projection Principle developed here accommodates the diversity of cases. Finally, by bringing together projective geometry and possible-world semantics, I’ll show that the Projection Principle offers a starting point for a systematic semantics of pictures.

While the Projection Principle is not meant as a full theory of depiction, I’ll argue that it has distinct advantages over many of the general approaches to depiction found in the contemporary philosophical literature. These include accounts which ground depiction in resemblance, similarity, or isomorphism (e.g. Peacocke 1987; Budd 1996; Hopkins 1998; Abell 2009; Blumson 2014), as well as those which seek to analyze depiction in terms of the perceptual responses that pictures elicit (e.g. Schier 1986; Wollheim 1987; Walton 1990; Lopes 1996). The Projection Principle diverges from each in important ways.

Section 1 introduces the Projection Principle: its content, motivations, and significance. Section 2 shows how the principle accommodates the variety of systems of depiction. Section 3 provides a formal restatement the principle within framework of possible-world semantics. Section 4 describes the semantic contribution of the Projection Principle using the concept of feature maps, derived from vision science. Section 5 contrasts the principle with extant proposals in the literature, including other projection-based accounts, resemblance and isomorphism theories, and perceptual theories of depiction. Finally, Section 6 explores the range and limits of a projection-based analysis, with attention to caricature, stylization, and inconsistent geometries. Section 7 is a conclusion.
In 1860 a visitor to the city of Timbuktu, Mali, published this engraving, which I’ll call Picture A. It depicts a collection of buildings as laid out in a loose grid; it depicts two people in the foreground as sitting down; it depicts what is likely the Djinguereber Mosque in the background as having a certain angular shape; and so on. What objects and individuals the picture depicts, and what properties and relations it depicts them as having, are all reflections of the picture’s CONTENT. The content of a picture corresponds to what’s happening in it, or the the situation it represents, which in turn determines the set of conditions under which the picture is accurate. Officially, I’ll say that pictures EXPRESS their contents, or informally, that they “depict” or “represent” their contents. The content of a picture in this sense should be distinguished, at least conceptually, from the perceptual content that the picture elicits from viewers; ultimately I will argue that they regularly come apart.

By PICTURES here I mean a broad class of public sign whose representational properties are in some sense distinctively visual. They are characterized in part by their 2-dimensional spatial

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1 The artist was probably Dieudonné Lancelot. See: commons.wikimedia.org/wiki/File:Barthtimbuktu.jpg.
2 One should therefore distinguish the present account from theories like those of Nanay (2015) or most accounts from psychology (e.g. Rogers 1995; Ittelson 1996; Koenderink and Doorn 2003) whose subject matter is the content of the perception of pictures, rather than the content of pictures themselves.
structure, and in part by their content, which, as I’ll argue, takes the form of a 3-dimensional space anchored at a viewpoint. Canonical examples of pictures include architectural and engineering drawings, figurative paintings, courtroom sketches, medical illustrations, and photographs.

The task for a semantics of pictures is to determine the rules, if any, by which pictures, in context, may be associated with the contents they express. There are many questions one may address in the service of a pictorial semantics. Philosophical theories of depiction have tended to focus on how and why pictures depict one object rather than another, or none at all, or why they attribute high-level properties like being a person, being a plant, or sitting down. (e.g. Schier 1986; Walton 1990; Lopes 1996) But such an account is incomplete unless it reckons with the distinctive geometry of pictorial content, and its intimate relationship to viewpoint. It is this, the underlying structure of pictorial space, which is the focus of this essay.

Though pictorial content is reflected in facts about what objects a picture is of, and what properties it depicts these objects as having, no single such fact captures the full spatial content of a picture. Picture A does not merely depict the two people in the foreground as sitting down, it also depicts them as having specific shapes, orientations, and locations; and it depicts every other object in its content as having specific orientations and locations relative to these two figures. Thus scholars have found it natural to conceive of the content of pictures as pictorial spaces: three-dimensional spatial arrays populated with individuals, properties, and relations. (Howell 1974; Wollheim 1987; Rogers 1995; DeLoache, Pierroutsakos, and Uttal 2003; Koenderink and Doorn 2003) A pictorial space can be said to realize a particular concrete scene to the extent that the scene conforms with the content of the space. (The concept of a scene used here will be elaborated shortly.) Pictorial space in this sense can be thought of as determining accuracy conditions: a pictorial space is accurate or inaccurate at a scene to the extent that the scene is a realization of that space.

As many authors have noted, what is distinctive of pictorial spaces is that they are, in some clear sense, perspectival (see e.g. Budd 1996; Hopkins 1998; Gregory 2013; Casati and Giardino 2013). That is to say that the objects and properties which inhabit pictorial space are all located only relative to a central perspective or point of view. The viewpoint-relativity of pictorial space

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3Officially I’ll define a picture as a metrically organized, colored, 2-dimensional array, where the array may be discrete or continuous depending on the system. Each element in the array corresponds to a point (or pixel) on the picture plane, and every such element is assigned a color (or color-like property). In the text, I’ll also use the term “picture” to refer to token concrete objects which exemplify such structure, disambiguating when necessary. While considerable idealization is involved in the passage from the concrete features of a picture token to the geometrical structure of its type, here I assume for simplicity that picture tokens realize structural types in some more or less direct way, and confine my attention to uncontroversial cases.

4A caveat for the case of photography: because the process by which photographs are produced is to such a high degree mechanical, reasonable questions have been raised about whether they should count as representations at all (Scruton 1981). Here I will proceed on the assumption that photographs are representations: they are pictures, they express content, and their content encodes conditions of accuracy. But the inclusion of photographs is not central to my theoretical aims.

5See Peacocke (1992, p. 62) for analogous remarks about perceptual space.
is exhibited in a variety of ways. For example, it arises in relations of depth: Picture A depicts the Djinguereber Mosque as being further from the viewpoint than the men in the foreground. And crucially for this essay, it is reflected in properties of direction: thus Picture A depicts the minaret of the Djinguereber Mosque as angularly above the viewpoint, and the man in the foreground as below it; and it depicts the pigeons on the roof as to the right of the viewpoint, and the men on the roof as just to the left. Critically, such directional relations are not limited to crude cardinal directions, but appear to locate every part of every depicted surface in a quantitatively distinct angular direction relative to the viewpoint.

The informal notion of viewpoint at work here can be regimented with a geometrical one. Henceforth, by VIEWPOINT, I mean a particular oriented location in space and time, in a particular possible world. The concept of an “oriented location” is definable within the framework of projective geometry, a matter I turn to shortly. The notion of viewpoint at work here need not be tied to any real or perceived viewer, or even an imagined one. It is simply an abstract, spatio-temporal index, what John Hyman (2006) has called an “objective eye”. Whereas viewpoints are token oriented locations, it is also useful to identify GENERAL VIEWPOINTS as types of oriented locations; they correspond to the origin points for the angular relations which make up pictorial spaces. All other objects in the pictorial space are given their position in relation to such a general viewpoint, but the general viewpoint itself is not tied to any particular location at a particular possible world. Instead, a given general viewpoint may be realized by any number of (token) viewpoints.

Earlier I said that pictorial spaces are realized by concrete scenes. I can now define a SCENE more precisely as a pair containing (i) a possible world and (ii) a viewpoint located in that world—what I’ll call, together, a VIEWPOINT-CENTERED world. A pictorial space is realized at a scene just in case, when the general viewpoint of the space is aligned with the token viewpoint of the scene, the positions of objects and properties specified by the pictorial space thereby correspond to the actual locations of those objects and properties within the scene. Note that any given pictorial space can be realized by indefinitely many scenes. This is because pictorial spaces are always partly indeterminate— if nothing else, they are silent with respect to occluded objects and objects outside of the picture plane— while scenes are themselves perfectly determinate. This conception of pictorial spaces as realized by sets of possible scenes will provide an important theoretical tool in laying out the semantic principles to come.

Returning now to my central question, how do two-dimensional picture surfaces express the rich, three-dimensional structure of pictorial spaces? By way of an answer, one popular strategy is to analyze the relationship between a picture and its content in terms of relations of resemblance or isomorphism; another casts visual perception itself as the representational glue between a picture and its content; I discuss these views in Section 5. The present view contrast with each. Joining Hyman (2006), Kulvicki (2006), Greenberg (2013) and others, I propose instead that it is geometrical

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6Thanks to an anonymous reviewer for this point.
projection which forms the semantic backbone of the correspondence between a picture and it’s space.

To begin, GEOMETRICAL PROJECTION describes any one of a family of methods for transposing a three-dimensional scene onto a two-dimensional picture plane via a system of spatial relations established relative to a viewpoint. The result is a picture: a determinate arrangement of line and color projected onto the picture plane. There are many different types of projection, a theme I return to in the next section. For now, a simple type of PERSPECTIVE projection is illustrated below. Here we begin with a concrete three-dimensional region of spacetime (possible or actual), which I’ll think of as a possible world. In the example below, the world contains only a cube. Next, a PROJECTION SOURCE is located within the space of the world. A projection source here is thought of simply as a geometric point in space and time. This in turn defines a system of PROJECTION LINES, which link each point in the world to the source. Finally, a PICTURE PLANE is introduced into this spray of lines, and they are used to map spatially distributed features of the scene back to surface features of the picture plane itself—in this case, the lines of the line drawing. The result of such a projection displayed at right below.

With these tools I can now precisely define the concept of viewpoint introduced above. A viewpoint is composed of a pair of indices, the first of which gives the spatio-temporal location of the projection source, and the second the spatio-temporal location of the picture plane. Together these make up what I have called an “oriented location,” where the projection source fixes the location, and the position of the picture plane relative to the projection source determines its ori-

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7The following presentation of the idea of projection draws from Dubery and Willats (1972), Sedgwick (1980), Willats (1997), and Greenberg (2013) among others.

8Note that the projection lines in the diagram below are only a representative sample of the full array.

9Note that the temporal location of the source and plane must be the same. Also, the picture plane itself must define at least up-down and left-right directions; front and back may be defined by the relative position of the projection source.
entation. (The notion of general viewpoint introduced above can be defined as the pair of an abstract point and picture plane, specified only in terms of their positions relative to one another.) Given a world and a viewpoint of this kind, a method of projection will always deliver a picture. In addition, it is viewpoints in this sense which figure in the formal definition of a scenes as viewpoint-centered worlds; thus methods of projection can be thought of as recipes for deriving pictures from scenes.

I turn finally to the Projection Principle itself. At least as early as Alberti’s 1435 monograph On Painting, scholars have recognized a connection between depiction and projection. But methods of projection have generally been cast as idealized techniques for constructing pictures from scenes. In the present account, the role of projection is reversed, now understood as an interpretive principle for determining pictorial spaces from pictures. Rather than a method for producing drawings, projection is enlisted as a semantic constraint on pictorial content.

The specific idea of the Projection Principle is that a picture can only express a given space if the picture itself could be a projection of that space. Thus, in order to correctly interpret a picture, we must treat it as if it were a projection of its content. Following Hyman (2006) and Kulvicki (2006), projection here is understood as a purely geometrical relation between a scene and a picture plane, not an actual process for producing a picture. A picture may be the projection of its pictorial space in this sense, even if the token picture and the objects in its content never stood in the relevant physical relation, or nothing took the role of projection lines or viewpoint. Thus, according to the Projection Principle, to work out a picture’s content is to figure what space it purports to be a projection of, but no restriction is put on the actual causal history of the picture.

The Projection Principle therefore suggests important points of similarity and difference between pictorial representation and the causally grounded phenomena of so-called “natural meaning” or “indication.” In the analysis of Leyton (1992), natural objects like shadows, tree rings, or footprints are traces of causal processes; to extract information from such an object is to work backwards from the trace to its causal source. The case of shadows is especially relevant, since shadows themselves are physical projections of three-dimensional objects. A shadow’s shape carries information about the shape of the object that cast it; to extract this information from a shadow is to determine what kind of object must have caused it, given the laws of optics. On the present view, pictures carry content in a manner analogous to the way shadows carry information. To calculate a picture’s content is to work how it must have been produced—only, the notion of “production” here is not that of causal production, as in the case of shadows, but of geometrical projection. In this sense, pictures are the abstract traces of the contents they express.

In order to formulate the Projection Principle precisely, two complications must be addressed. First, I said above that the principle holds that pictures are projections of their spaces; however, my preferred notion of projection applies to scenes, not spaces; and it’s not clear that the idea of taking the projection of a pictorial space can really be worked out in detail. Instead, in its official

§1 The Projection Principle
articulation, the Projection Principle holds that, given a picture and the pictorial space it expresses, for every scene which realizes that space, the picture is a projection of that scene. Thus the principle indirectly constraints what space a picture may express by directly constraining the set of scenes which may realize such a space. This amounts to a constrain on content, since pictorial spaces whose realizing scenes do not project back to the picture are thereby ruled out.

The second complication is brought about by the existence of different forms or systems of depiction, a matter I return to in the next section. As I’ll argue there, the Projection Principle analyzes such systems in terms of different forms of projection. Thus, officially, the Projection Principle claims that, for a picture to express a pictorial space as content relative to a system, that space must project back to the picture itself, according to the method of projection characteristic of that system. Putting these ideas together, we can now informally state the Projection Principle in schematic form. (A formal statement is postponed to Section 3.)

**The Projection Principle (informal)**

If a picture $P$ expresses pictorial space $C$, relative to system $S$, then for any scene $E$, centered at viewpoint $V$, which realizes $C$:

$P$ is a projection of $E$ from $V$, via the method of projection determined by $S$.

The contribution of the Projection Principle to pictorial semantics is at once both sweeping and minimalist. First, consider its coverage: it correctly adjudicates attributions of content for all examples selected from core cases of pictorial representation; and it does so with essentially unlimited specificity. For now we may think of these adjudications as purely negative: for any given picture, the principle correctly predicts which contents that picture could not express. For example, given Picture A, apparently depicting a complex of buildings laid out in space, the Projection Principle allows us to rule out innumerable bizarre interpretations. We know from the Projection Principle alone that Picture A does not express a space consisting only of an empty room, a seaside view, a herd of elk, or infinitely many other spurious interpretations. Indicative of its predictive power, the Projection Principle also allows us to make arbitrarily fine-grained assessments of inaccuracy as well. For example, by the Projection Principle, we know that the images below have different contents; we also know that Picture D could not depict a scene containing only a (perfectly regular) cube, while Picture C could— with the same result, of course, for even more subtle differences. The quantitative precision of such assessments is one of the notable virtues of the Projection Principle as a tenet of pictorial semantics. As I’ll argue in Section 5, alternative resemblance- or isomorphism-based accounts of depiction issue in only very course semantic constraints as compared to those demonstrated here.
Though powerful, the constraint which the Projection Principle imposes is in an important sense only skeletal, and does not constitute a complete pictorial semantics. The point is made vivid in the diagram below. Picture E seems to express a space inhabited by a cube, yet the picture itself can be projected from innumerable many other non-cube shapes, as illustrated. As a consequence, the Projection Principle cannot rule out these deviant scenes as possible interpretations of Picture E. And if such non-cube shapes cannot be ruled out by the Projection Principle, then the principle alone cannot explain why the property of being a cube seems to figure in E’s content. The point can be extended: for the same reasons, the principle alone cannot explain the attribution of high-level properties, like being a person or being a plant, nor even relatively low-level spatial properties like depth, shape, or texture. Yet at least some of these are clearly part of a picture’s content, which is why the Projection Principle falls short of a complete theory of depiction.

Ultimately, the Projection Principle merely requires the picture’s content include those (quite abstract) spatial properties had in common by the myriad possible bizarre scenes which could project to the same picture. As I’ll demonstrate in Section 4, these spatial properties in fact constitute a relatively natural class: they consist of all (and only) the angular locations of all low-level features like edge, surface, and color featured in the picture’s content. I call this a picture’s PROJECTIVE CONTENT (and formalize the idea in Section 4).\(^\text{10}\) So while the Projection Principle cannot explain the attributions of reference or high-level properties we normally associate with pictures, it does explain the attribution of projective content, which in turn specifies the basic angular layout.

\(^{10}\)Projective content is comparable to Kulvicki’s notion of barebones content (Kulvicki 2006, ch. 3). See Section 5 and the appendix for discussion.
of rudimentary features in pictorial space.\footnote{With the addition of background assumptions, projective content carries somewhat more communicative value. For example, a picture that could accurately depict a cube may also accurately depict infinitely many different irregular shapes. But if we assume that it accurately depicts a Platonic solid, then we may conclude determinately that the object in the scene depicted is a cube. Introducing assumptions like this constitute one proposal about how projective content is fleshed-out, but is not, I think, the final answer.}

We can now see why the explanatory significance of the Projection Principle runs deeper than merely specifying one ingredient of pictorial content among many. For the content it contributes describes the spatial substructure necessary for any further enrichment of pictorial content. Given a picture, whatever it’s overall content, it only expresses that content partly in virtue of expressing its projective content. For example, a picture might depict the individual Ada Lovelace as exhibiting the property holding a fan. But it only depicts Lovelace in virtue of depicting her at a particular location in visual space; and this, only in virtue of the more basic depiction of surfaces and colors arranged in space; and these only in virtue of the even more basic angular array of features that make up projective content. In general, for any given bit of high-level content \( C \) (which may be only a part of a picture’s total content), it is impossible for a picture to express \( C \) without also expressing some projective content.\footnote{The reverse is not true: it is not the case that for any projective content \( R \), it is impossible for a picture to express \( R \) without also expressing some high-level content. For example, an image containing only static-like texture, or an all-white image, would have projective content, but plausibly lack any significant high-level content.} It is in this sense that projection grounds pictorial representation: without the projective content imposed by the Projection Principle, no other aspects of pictorial content would get footing.

In providing a structural foundation for pictorial content, the Projection Principle not only describes pictorial representation, but also, I believe, characterizes one of its essential features. To be a pictorial representation is, in part, to have a semantics grounded in projection. I have already shown how this claim applies to the core case of perspective line drawing. In Section 2, I’ll further substantiate the claim about pictorial representation \textit{in general} by showing how the Projection Principle can be extended to a wide range of drawing systems far beyond perspective drawing. And in Section 6, I’ll argue that the same basic idea even extends to more elastic pictorial forms, including systems of caricature, stylization, and inconsistent geometry. What all forms of pictorial representation have in common, I hold, is that they are ultimately grounded in projection.

This fact distinguishes depiction from both linguistic and diagrammatic representation. On one hand, linguistic representation is founded on arbitrary lexicons paired with compositional rules for interpreting tree-like syntactic structures—nothing like the two-dimensional structure of pictures or their projective interpretation. On the other hand, diagrammatic representations, like Venn diagrams, XY graphs, directed graphs, pie charts, and many others, while manifestly \textit{spatial}, do not express their content in a way that is associated with a spatially-anchored viewpoint, and are not understood in terms of geometrical projection (Shin 1994; Casati and Giardino 2013). Projection, in short, appears to be a central mark of the pictorial.
But why should pictorial representation, as opposed to linguistic or diagrammatic expression, be grounded in projection as I have suggested? I conclude this section by outlining an answer to this question in two steps: first, pictorial representation depends on core capacities of human vision; and second, vision itself is grounded in projection (albeit of a somewhat different kind than that of pictorial representation). In short, depiction is projective because vision is.\(^\text{13}\)

To develop this idea, I assume without argument a broadly computationalist view of perception, one that is widely held, especially among cognitive scientists, but also contested by some philosophers. (Advocates include Palmer (1999), Burge (2010), and Frisby and Stone (2010).) According to this account, the visual system functions to continually compute the layout of the scene before it, on the basis of the evolving retinal image. The results of this computation are perceptual representations—perceptual states which describe the spatial environment before the perceiver, and which may be correspondingly accurate or inaccurate.

I begin with a claim which should be uncontroversial: the interpretation of pictures, at least among core cases, draws upon cognitive resources that largely overlap those deployed in normal (non-pictorial) episodes of visual perception. Not only is this what we actually find (e.g. Rogers 1995; Cutting 2003), it is what we should expect. As Wollheim (1987) has noted, the phenomenology of normal visual experience is strikingly similar to the experience of looking “into” a picture. And the interpretation of pictures clearly relies on the same kinds of cues which visual perception exploits to extract information from the retinal image. To name just a few illustrative examples, converging lines, occluded shapes, and texture gradients are all taken as depth cues in both the visual and pictorial domains (Solso 1996, ch. 7).

Not only do picture interpretation and vision in fact overlap at a cognitive level, I claim that they must. To be pictorial, a system of representation must be based in vision. In particular, the computations necessary to apply that system’s interpretive rules must be structurally similar to the computations executed by the visual system in its normal operation. Contrast this view with that of perceptual theories of depiction (discussed in Section 5), which hold that the content of a picture can be identified with the content of a particular perceptual state (the state elicited by looking at the picture). I reject this simple identity of pictorial and perceptual content. The relationship which depiction bears to vision is more flexible and holistic: the computations required by depiction must be analogous to those required by vision—even if the resulting states express markedly different contents. An analysis of depiction based in projection precisifies a central feature of this analogy. As I’ll argue, the interpretation of pictures must be like visual computation insofar as both are grounded in projection from a viewpoint; yet the form and structure of this projection need not be the same in the two cases. I explain each of these points in turn.

According to the computationalist understanding of vision, the specific function of the visual

\(^{13}\)Here I’ll treat the terms “perception,” “visual perception,” and “vision” as roughly synonymous. I’ll also use the term “visual system” to refer to the system of visual perception, neglecting visual imagery and visual memory. Throughout I’ll assume that the perceptual capacities under discussion are those of humans with normal vision.
system is to generate an estimate of the kind of scene that must have produced the retinal image. (Burge 2010, pp. 308-315; Marr 1982, pp. 32-38) But since, for any given image, infinitely many scenes could have caused it, there is no unique solution to this problem. This is the so-called “inverse problem” of vision. The visual system overcomes this problem by making a best guess, informed in turn by implicit heuristic assumptions about the environment (Palmer 1999, pp. 80-85). And the most basic such assumption is that the retinal image itself is the product of the optical projection of light through the lens of the eye (Rock 1983, p. 324).14 Without this background constraint, further implicit assumptions about regularities in the environment cannot be reliably related to the retina image. Thus encoding the laws of optical projection is the ground upon which more refined visual computations must be built.

The methods of projection which characterize depiction have in common that they all can be derived, in one way or another, from this original case. Indeed, the very concept of geometrical projection may be thought of as an abstraction and idealization of the behavior of light as it reflected to the eye. As art-historian Erwin Panofsky observed in 1927, “Exact perspectival construction is a systematic abstraction from the structure of this psychophysiological space” (Panofsky 1991, p. 30). The retinal surface is modeled as the picture plane; the focal point defined by the lens of the eye is modeled as the projection source; and the ray-like behavior of light which links the scene to patterns of activity in the retina, now become the straight lines of projection. Just as light carries information about surfaces that it has reflected off of to receptor cells in the retina, lines of projection transfer features of the scene (like edges) to features of the picture plane (like lines).

I propose that interpreters of pictures face an inverse-problem analogous to that facing the visual system, and an analogous first-pass solution is deployed; for pictures, this solution is the Projection Principle. In calculating the content of a picture, the interpreter must (explicitly or implicitly) assume that the picture is derived by projection from the space it expresses, just as human vision always (explicitly or implicitly) assumes that the retinal image is derived from the scene before it by the laws of optics. The interpreter of a picture is not trying to derive the scene that must have, as a matter of fact, causally produced the image. Rather, her aim is to derive the scene from which the picture is purported to have been a projection, in virtue of the communicative commitments of the artist. Nevertheless, if the Projection Principle is right, the computations performed by an interpreter of pictures are at least in this respect analogous to those performed by the visual system in the process of “interpreting” the retinal image.

Still, vision and depiction are not the same: different methods of projection cleave more or less closely to the behavior of light projected to the eye. Following a distinction suggested by Willats (1997), we may roughly distinguish optical and non-optical methods of projection. Optical meth-

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14 The sense in which such laws are “assumed” by the visual system is an open question; it is unlikely that they are actually represented, but they may well be encoded in the normal operation of the system itself. Either way, the inverse problem is intractable unless assumptions about the projective behavior of light are in some way encoded.
ods of projection cleave as much as possible to the physical behavior of light, both in terms of the geometry of the projection lines and in the encoding of color and shading.\textsuperscript{15} Such projections are characteristic of realist, linear perspective painting styles, and many forms of color photography; they produce the kinds of images which, under suitable circumstances, can become \textit{trompe l’oeil}. But there is a proliferation of \textsc{non-optical} systems as well. Many have argued that linear perspective itself—the form of projection which I have used as my central example thus far—though similar to optical projection, diverges in its geometry. In the next section, we will see examples of projective geometry, such as oblique projection, which diverge even more extremely from the optical norm. As for marking, even black and white photography is a departure from optical projection. More dramatically, line drawing involves the registration of edges as lines, quite unlike the information registered by the retina.\textsuperscript{16} Correspondingly, no picture in a non-optical system could serve as a \textit{trompe l’oeil} depiction of its content.

Systems of projection diverge from in-built visual rules for a variety of practical reasons. These include limitations of artistic skill, ease of construction and ease of use or interpretation, communicative or perceptual effect, the need for standardization, and so on (Giardino and Greenberg 2015). Thus the methods of projection investigated in this essay are based upon and crystalize (the visual system’s assumptions about) the projective behavior of light to the eye. But because they are defined directly in terms of geometry, and not in terms of actual optical laws or physiology, they are allowed to disengage from the optical standard freely, in response to the practical requirements of human use. It remains an open question exactly how non-optical depiction (and for that matter, much optical depiction) is interpreted at a cognitive level. It’s clear that the visual system is deeply involved in picture interpretation. But current models of perception give us little sense of how non-optical systems are engaged by visual cognition.

From the perspective of human communication, the enlistment of vision as a mechanism for interpreting signals seems inevitable. The human visual systems is computationally powerful; all normal humans have one; and all normal human visual systems are very similar to one another. A signaling system which exploited these facts would have at its disposal a fast and reliable interpretive engine shared by most interlocutors. This alone would obviate much of the individual work of learning a communicative code, and much the collective work of coordinating on one. Systems of pictorial representation, I conjecture, are the product of this conflux of innate visual ability, practical demands, and the interpersonal drive to communicate. Geometrical methods of projection recapitulate the optical projection of light to the eye. In so far as these methods can be directly computed by the visual system itself, they will naturally be favored for use. But in so far as pictorial representation is subject to external constraints it may stray from the strictures of vision. Still, there is a limit: to be a form of pictorial representation, a representational system must

\textsuperscript{15} In fact, systems can be optical with respect to geometry, but not markings, and vice versa.

\textsuperscript{16} As I will discuss in Section 6, systems of stylization and caricature diverge even more starkly from optical projection.
ultimately be based in projection.

2 Systems of Depiction

Any account of pictorial representation must accommodate the central fact that depiction varies in form relative to what I shall call SYSTEMS OF DEPICTION, the pictorial analogues of languages (Gombrich 1960; Hagen 1986; Willats 1997). Not only do pictures belong to certain systems, the expression of content itself, as we’ll see, must be relativized to an operative system of depiction. Picture A, for example, is meant to be interpreted in the system of linear perspective line drawing with shading, while a typical realist painting would be evaluated relative to the system of linear perspective with optical color. In general, systems of depiction vary along a host of dimensions, including their treatment of geometry, line, light, and color. For my purposes, variation in the deployment of geometry is of special interest in what follows, because it gives rise to pictorial spaces with fundamentally different structures.

These points are illustrated by comparing two such systems. The first is the linear perspective system of depiction, the system of Picture A. While this system has ancient roots, it was first codified during the Renaissance, and now dominates contemporary mass media. Linear perspective is marked by the fact that objects at greater distances from the viewpoint are depicted by smaller regions on the picture plane, as in figure F below. As a corollary, parallel lines extending away from the viewpoint are depicted by converging lines on the picture plane, as in figure G. Both features are also apparent in Picture A.

PARALLEL systems of depiction, by contrast, are less standard in mainstream media, but are the norm in architecture, engineering, and other technical fields, and are commonplace in the art of classical East Asia and ancient Egypt. There are many types of parallel system, including isometric, axonometric, and orthogonal projection systems. The same features which characterize depiction in linear perspective distinguish it from parallel depiction. In parallel systems, even as objects move farther from the viewpoint, they are depicted by regions of the same size on the picture plane, as in figure H. Thus parallel lines extending away from the viewpoint are depicted by

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17 Linear perspective is often simply called “perspective.” Linear perspective is to be distinguished from curvilinear perspective—the sort of image produced by a fisheye lens.
18 See Dubery and Willats (1972) for a clear discussion of the distinctions.
parallel lines on the picture plane, as in Picture I. Parallel systems give rise to a kind of unsituated “god’s eye view,” though such pictures are still perspectival, since they clearly depict their subjects from a particular direction.

Pictorial systems like these are not merely styles—guidelines for construction with no semantic consequences. Rather, they embody genuine rules of interpretation which associate different kinds of pictorial space to the pictures in their domains. This why pictures are not merely understood as the products of different systems, but the expression relation pictures bear to their contents is itself relativized to a system. As illustration, suppose we isolate Picture J without specifying the intended system of depiction, and ask after its content. Relative to the linear perspective system, it may express a space containing a cube, but relative to a parallel system, it can only express a space containing an irregular solid. This is because, in linear perspective, the converging lines on the picture plane may be interpreted as representing the parallel edges of a cube; but in parallel systems, converging lines can only be interpreted as representing converging edges. Thus different systems of depiction determine different ascriptions of content, in part by associating pictures with spaces of fundamentally different structure.¹⁹

According to the Projection Principle, systems of depiction affect content by fixing the underlying method of projection. So to interpret a picture as part of a perspective system is to treat it as if it were produced from the corresponding method of projection. And the interpretive variation between parallel and linear perspective systems can be attributed to an alternation between corresponding methods of projection. As noted in the previous section, such system-relativity is already built-in to my original statement of the Projection Principle: if a picture P expresses content C relative to system S, then for any scene E compatible with C, P must be a projection from E “via the method of projection determined by S.” Here I provisionally assume that every system of depiction determines a unique method of projection, what I will call its characteristic method of projection.²⁰ Then the Projection Principle claims (roughly) that for a picture to express a pictorial space as content relative to a system, that space must project back to the picture itself, according to the method of projection characteristic of that system.

In fact there are many different methods of projection, all following roughly the same recipe. The method illustrated in the previous section is known as linear perspective projection (for short, perspective projection); it is characterized by the fact that the projection source is a point, to which all projection lines converge. An alternative method, known as parallel projection

¹⁹The same point can be made with respect to the difference between systems of black and white line drawing and systems of color line drawing. Relative to a system of black and white line drawing, Picture J might depict a solid of any color or shade—its content is indeterminate in this respect. Relative to a system of color line drawing, it must depict a white solid on a white background.

²⁰This isn’t always the case. There are systems of depiction which involve more than one method of projection. Generally such systems fall under the rubric of “impure projection,” discussed in Section 6.
replaces this point with an oriented plane. Then the projection lines, rather than converging on a single point, all run perpendicular to the projection source, hence parallel to one another. An example is illustrated below. In this case, the projection source and picture plane are themselves parallel, but this is not required. The familiar examples of axonometric, isometric, and oblique projection are all species of parallel projection, derived in part by varying the relationship between projection source and picture plane. Even though the projection source of parallel projection is a plane, I still refer to the combination of projection source and plane as a viewpoint since it undeniably provides a type of spatial perspective on its subject matter. Still other methods of projection can be defined by varying the structures of the viewpoint, picture plane, projection lines, and their relative relationships.

Each method of projection specifies a different way to construct a picture, and these methods produce images with visibly divergent geometries. These differences in turn reflect the interpretive contrasts we noted earlier, for example between perspective and parallel systems of depiction. To interpret a picture as part of a perspective system is to treat it as if it were produced from its content by a method of perspective projection; as a consequence, converging lines in the picture may

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21Hagen (1986) offers an alternative analysis, according to which the projection source of parallel projection is actually a point, but it is located at infinite distance from the scene.

22Thus there are not only various species of parallel projection, but still other forms of projection altogether, like curvilinear perspective, reverse perspective, various globe projections, and so on. In the text, I discuss methods of projection where the projection lines are themselves straight lines. But in a prominent alternative class of projections, lines of projection instead closely recapitulate the behavior of light rays, reflecting and refracting in the environment before reaching the picture plane. Such “optical” methods of projection are the norm in realist painting, computer animation, and, of necessity, photography. The construction of images in modern 3D animation uses a method of rendering known as “ray tracing”—essentially tracing the complex path that light would realistically take in context.
indicate parallel edges receding from the viewpoint. By contrast, to interpret the picture as part of a parallel system is to treat it as if it were produced from its content by a method of parallel projection; thus converging lines in the picture can only represent converging edges in the content.

Among the most dramatic variations in the use of projective geometry arises for the case of maps. In general map projections use a variety of techniques for projecting curved surfaces (at the limit, spheres) onto 2D pictures planes. Such methods variously place the projection source above, below, or even within the sphere, in order to project it onto a facing picture plane (Robinson et al. 1995). Though not intuitively “perspectival” in quite the same way as traditional pictures, maps nevertheless embody “views” of the landscape they canvas by depicting one or another part of the sphere at the center of the map, in a given orientation, and by, for example, depicting the surface of the sphere, rather than its interior, or a cross-section.

We’ve now encountered some of the geometrical variation among methods of projection; but, as advertised, they also vary widely in their treatment of line and color. Indeed, following Willats (1997), methods of projection may be divided into two components. The first component, specified by a projection condition, establishes the systematic spatial relationship between points in the scene and points on the picture plane, as embodied by projection lines. Perspective and parallel projection represent two ways of filling out the projection condition. The second component, specified by a marking condition, associates properties of the scene with types of mark on the picture plane, following the spatial distribution specified by the first stage. A method of projection is then defined as any combination of projection condition and marking condition.

Thus far, all of the pictures discussed have been line drawings—a particular type of marking condition. Very roughly, line-based methods of projection dictate that visible edges in the scene are mapped to lines on the picture plane. By contrast, in methods of color projection, characteristic of painting and photography, colors in the scene are mapped to suitably related colors in the picture. (“Color” here stands in for any color-like property, such as reflectance or illumination.) These and other variations—by no means exhaustive—are illustrated below.

![Diagram of projection methods]

24 For work on the theory of line, see Kennedy (1974), Willats (1997), Palmer (1999), and DeCarlo et al. (2003). For discussion of color and other “optical” methods, see Willats (1997, ch. 6). With these tools, even photographs can be conceived as the products of projection. Most photographs exploit a projective geometry approximately like linear perspective, and an optical system of marking loosely analogous to the “color-to-color” method illustrated below.

§2 Systems of Depiction

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Thus, by bringing together results from research programs in art history, technical drawing, geometry, and computer graphics, scholars like Dubery and Willats (1972), Hagen (1986), Willats (1997), and Durand (2002) among others, have argued that an impressive range of art historical “styles” can be understood as arising from the same basic projective ingredients as those reviewed here. This speaks strongly to the validity and scope of the Projection Principle. (To be sure, there are still other systems of depiction which cannot be analyzed directly in terms of projection; I return to this theme in Section 6.)

Projection is clearly a flexible analytical tool. But is it too flexible? There is a danger that “projection,” unless constrained, cannot be a sufficient basis for a sign system to be depictive. The problem is that there are methods of projection which are clearly non-pictorial. I have characterized projection as a mapping from a 3D scene to a 2D picture plane via a systematic set of spatial relations defined relative to a viewpoint. And I have given concrete examples of core cases. But one can imagine a spectrum of cases, deviating from this core but adhering to the general description, which seem less and less pictorial at each step. Consider, for example, projections which bend and distort their subject matter in unusual ways, in the manner of a fun-house mirror. Such images seem essentially pictorial, albeit in a somewhat degraded way. But more extreme cases resist inclusion: consider a mapping from scenes to pictures that exploits a specific configuration of seemingly scrambled projection lines. The results of such a projection would look like static or confusing swirls of color. Still, it could be a projection: the viewpoint for such a method might take the form of a highly fractured surface; and it might use the same scrambled configuration for every picture, hence qualifying as establishing a “systematic” spatial relation. Nonetheless, the products of such a method would resist classification as “depictions.”

In fact, I doubt that there are fully general necessary and sufficient conditions for being a system of depiction, especially if these conditions are to be specified in geometrical or structural terms. The range of variation among systems of depiction is too great, and new systems are constantly being introduced into use which extend, bend, or contravene the structural standards that were formerly established. Instead, as I suggested in the last section, systems of depiction are identified in part through their functional association with vision. A method of projection can be the basis for a system of depiction if it is derived from projective rules encoded by the visual system, where “derivation” is purposefully left open-ended, allowing for step-wise deviations of both overall geometry and marking. In essence, the range of possible systems of depiction corresponds to those projection-based systems that harness central aspects of visual cognition, or are systematically related to systems that do. This characterization doesn’t require that non-optical methods of projection actually be computable by the visual system, only that they be related to sys-

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25The theoretical situation is akin to one familiar within the philosophy of language. Various formal-logical definitions of the concept of language are available, but many of these are so abstract, complex, or bizarre that they resist the moniker “language” in any straightforward sense. Theorists have tended to respond (e.g. Chomsky 1965; Lewis 1975) by identifying some of these formal systems as the humanly usable languages.

§2 Systems of Depiction
tems that are. Still, one should expect systems to proliferate which can be helped along by visual processing, and we have reason to think that our core cases—like linear or parallel projection—are like this.26

3 Pictorial Semantics

A semantics of pictures should determine the rules, if any, by which pictures may be associated with the contents they express. I've argued that the Projection Principle is one such rule. In this section, I develop this claim by bringing together two sets of formal tools: possible worlds semantics, on one hand, and projective geometry, on the other. Besides yielding a more precise understanding of the Projection Principle, such a semantics makes the principle commensurate with semantic rules familiar from philosophy of language and linguistics. Indeed, I hope this discussion makes it clear that depiction, at least for the aspects under review, is every bit as systematic as linguistic expression, even if the rules in question diverge sharply.

I begin by regimenting the expression relation between pictures and contents. I assume that a picture expresses content only relative to a CONTEXT and a SYSTEM OF DEPICTION.27 I include context here for completeness, but it won't play a central role in what follows. Formally, I'll model the expression relation as an interpretation function which takes as arguments a picture, system of depiction, and context and returns a content.28 The content expressed by a picture \( P \) relative to a system \( S \) and context \( c \) is written \([P]_{S,c}\). (Officially, one may that think of \( S \) merely as the system of \( c \), in which case the representation here is redundant; I assign \( S \) an explicit parametric role for expository convenience.)

Next I wish to model pictorial content within the framework of possible-world semantics. Here I enlist the concept of a viewpoint-centered world or scene, defined as the pair of a possible world and a viewpoint, introduced in Section 1. Then, following Ross (1997) and Blumson (2009b) we may model a picture’s content as a set of scenes.29 Formally, such a set may be thought as a relation

26 My proposal here bears some affinities to that of Kulvicki (2006, p. 76), since we both identify systems of depiction in terms of their relative proximity to a core class. But Kulvicki defines that class in terms a formal property he terms “transparency,” while I define mine in terms of visual cognition. The two ideas come apart because systems can vary in their degree of proximity to transparency, without varying in their degree of proximity to visual cognition, and vice versa. Systems like edge- and topography-based line drawing, for example, flagrantly violate transparency, but are closely related to visual cognition. The pursuit of transparency also pushes Kulvicki into weakening the projection analysis in a way that I criticize in the appendix.

27 In a final accounting, it may be necessary to relativize content expression to a broader suite of SYSTEMS OF INTERPRETATION, which capture the general rules by which picture types are mapped to all aspects of their content. Such systems are likely heterogeneous, including both visual and pragmatic mechanisms, as well as purely geometric and chromatic constraints (Kulvicki 2006; Giardino and Greenberg 2015). It is the last type of system which I term systems of depiction here.

28 Formally, a picture’s structure can be modeled as a triple \((P, d, c)\), where \( P \) is a set of points, \( d \) is a distance metric over \( P \), and \( c \) is an assignment of colors (or something like colors) to contiguous subsets of \( P \). The distance function \( d \) may be interpreted as a measure of absolute or relative distance, depending on system of depiction. Constraints on \( d \) ensure that the picture is a two-dimensional, rectangular plane-segment.

29 Blumson (2009b, §3-4) proposes that we define the content of pictures as sets of centered-worlds, conceived of as “ordered quadruples of orientations, locations, times and possible worlds.” I find it more natural to combine the elements

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between worlds and viewpoints: the content of a picture imposes a simultaneous constraint on the way the world is relative to a viewpoint, and the location of that viewpoint relative to the world.

This model of a picture’s content fits smoothly with the characterization of pictorial content as pictorial space anchored at a general viewpoint. This is because every such space determines a set of scenes— those pairs of worlds and viewpoints for which the viewpoint realizes the anchor of the pictorial space, and the world realizes the space itself. Of course, an arbitrarily selected set of scenes won’t correspond to a pictorial space in this way, as its members may be entirely disjoint. But those sets of scenes which are the contents of pictures will all coincide precisely insofar as the picture’s represented space is determinate, and diverge insofar as it is indeterminate.

It will prove illuminating to characterize the Projection Principle both in terms of its contribution to the content pictures express, but also as a part of the conditions under which a picture can be accurate. This bring the Principle in line with the familiar rules of linguistic semantics, which characterize content, but typically deliver truth-conditions (or conditions of satisfaction) as well.

My starting point is the notion of relative accuracy: the content of a picture is ACCURATE AT or INACCURATE AT a scene when the content is realized at, or compatible with that scene— much the way linguistic propositions are thought to be true or false relative to a world. Since contents are modeled as sets of scenes, the compatibility between a content and a scene is be modeled as the scene being a member of that set. Thus the content of a picture is accurate at a scene if and only if the scene is a member of that content. Two terminological remarks: first, for a picture’s content to “accurately depict” a scene is simply for that content to be accurate at the scene. Second, though it is contents which are accurate at scenes, we may speak in a derivative sense of a picture, relative to a system and context, being accurate at a scene, when its content is.

An important wrinkle here is that accuracy, unlike truth, comes in degrees; what degree of accuracy defines a picture’s accuracy-conditions? Here I help myself to the notion of maximal or PERFECT accuracy: the content of a picture corresponds to the set of scenes relative to which that content is perfectly accurate. Henceforth, by “accuracy” I mean perfect accuracy (or very near it); by “inaccuracy” I mean less than perfect accuracy. Providing an account of graded accuracy is left to future investigation.

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Accuracy here is meant to be the pictorial analogue of truth for sentences (Greenberg 2013, p. 224). A picture’s content is accurate at a scene when, to the extent that it represents things as being a certain way in that scene, that is the way those things are. Accuracy, in this sense, does not imply realism or closeness to reality. A black and white drawing and a color painting may each be perfectly accurate, albeit relative to different systems, though the experiences they elicit obviously differ in the proximity to normal perceptual experience. A full scale, working model might be closer to reality, in some sense, than a technical drawing, but both may be perfectly accurate. What matters for accuracy is the absence of misrepresentation—not the quantity or type of information represented. This notion of accuracy is not the exclusive denotation of the word “accuracy” in colloquial English, but it is arguably one of them.

Thus far I’ve described scene-based models for pictorial content and for accuracy. Using these tools, Projection Principle can be understood as a constraint on content, or equivalently, as a necessary condition on accuracy. The final necessary ingredient is a formalization of projection itself. But this flows naturally from the idea that methods of projection are algorithms for producing pictures, given a world and a viewpoint. I’ll say that every method of projection determines a projection function, a function from worlds and viewpoints to pictures; or equivalently, a function from scenes to pictures. Here I’ll refer to the characteristic method of projection for a system of depiction $S$ as an “$S$-projection,” and I’ll notate the corresponding projection function $\text{proj}_S(\cdot)$.

I turn now to the principle itself. In my original formulation, I construed the Projection Principle as an indirect constraint on what pictorial space a picture may express, via a direct restriction on what scenes could realize that space. I now understand pictorial spaces as sets of scenes, and the realization relation between a scene and a space as set membership. Then the principle requires that, for a scene to be a member of the content of a picture, the picture must be a projection of that scene:

**Projection Principle for Pictorial Content**

For any picture $P$ in system $S$, context $c$, and scene $\langle w, v \rangle$:

- if $\langle w, v \rangle$ is in the content of $P$ relative to $S$ and $c$,
- then the $S$-projection of world $w$ from viewpoint $v$ yields $P$.

Formally: if $\langle w, v \rangle \in [P]_{S,c}$ then $\text{proj}_S(w, v) = P$.

Put another way: the content of any picture is compatible with the set of scenes which project to the picture: $[P]_{S,c} \subseteq \{ \langle w, v \rangle \mid \text{proj}_S(w, v) = P \}$. The principle effectively narrows the range of scenes compatible with the content of a picture by ruling out exactly those scenes which fail to project to the picture, and allowing in all others.

A formulation in terms of accuracy is equivalent:

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§3 Pictorial Semantics
Projection Principle for Pictorial Accuracy

For any picture P in system S, context c, and scene ⟨w, v⟩:

if the content of P relative to S and c is accurate at ⟨w, v⟩,
then the S-projection of world w from viewpoint v yields P.

Formally: if \([P]_{S,c}\) is accurate at \(⟨w, v⟩\) then \(\text{proj}_S(w, v) = P\).

In both formulations, the contextual parameter on the left side of the formula does not appear on the right. This reflects the fact that, although pictures express their contents only relative to contexts, the Projection Principle itself describes the context-independent contribution of the system of depiction and the picture type to the determination of content. The Projection Principle thus captures part of the context-independent semantic competence one acquires by learning or internalizing a system of depiction.

These specific formulations of the Projection Principle should be thought of as templates, from which versions of the principle may be derived, by varying the details of its main clause, or adding further restrictions. For example, accommodations may be made for systems that employ wobbly lines or variation in scale, as well as for more complex methods of projection, restrictions on the configuration of viewpoint, salient spatial features of the world, and so on. The proper treatment of such phenomena likely require alterations to the letter of the constraints stated here, but the general principle remains the same.

By situating the Projection Principle within the framework of a possible-worlds semantics, I hope to have made it clear the principle itself is a genuinely semantic rule, in the sense that it articulates a precise, rule-governed constraint on the association between a pictorial sign and its content. While a semantics based on projection is obviously very far the semantic rules which ground linguistic meaning, both can now be seen to fall under the broader genus of semantic theories. I thereby align myself with the tradition of Goodman (1968) on sign systems, Shin (1994) on diagrams, Casati and Varzi (1999) on maps, Lascarides and Stone (2009) on gestures, Schlenker, Lamberton, and Santoro (2013) on iconic sign language, and many others who have shown that the science of semantics can thrive outside of the linguistic arena.

4 Projective Content

The introduction of a scene-based semantics helps clarify the sense in which the Projection Principle is a semantic constraint. But, as I noted in Section 1, the import of this constraint appears on its face to be negative: for any given picture, it rules out spurious interpretations of that picture. Yet its also possible to think of the Projection Principle as specifying a positive semantic contribution to pictorial content, what in Section 1 I called a picture’s projective content. In order to conceptualize (and visualize) projective content, a somewhat different set of formal tools is called
for, this time borrowing from the vision scientist’s notion of feature maps. A feature map is a two-dimensional spatial array (or “map”) for which different points in the array are associated with a various properties and relations (or “features”). A feature map is accurate at a scene when, for each point in the array, there is a point in the scene which instantiates the features associated with the array point. In this way, feature maps are also the bearers of content.

The Projection Principle has the consequence that every picture’s content includes, or is constrained by a certain kind of feature map, what I will call a projective feature map. A projective feature map assigns to each point in its array (i) an angular direction $\theta$ and (ii) an low-level environmental feature (or set of features) $F$. An example is illustrated below, for a map consisting of only four “points.” Crucially, each of the associated angles must be defined relative to a central viewpoint, as I’ll discuss shortly. A projective feature map is accurate at a scene if and only if, for each point in the array, there is a point in the scene located along the angle $\theta$ relative to the surface of the array which has the feature $F$.

To see how the Projection Principle associates pictures with projective feature maps, assume that for any scene in a picture’s content, that scene projects to the picture. As the diagram below illustrates, no matter what the world of that scene is like, each projection line will pass through the picture at a specific angle; these then become the angles that populated the feature map array. Next, the features associated with each point in the array correspond to those features of the environment which would project to the chromatic properties of the corresponding point in the picture. For example, in a system of line drawing, where edge-points project to line-points, every line-point on the picture plane will be associated with the feature edge in the corresponding point in the array.

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32 Feature maps are widely associated with visual representations in cognitive science. See Marr (1982, Ch. 4) and Tye (2000, Ch. 5) for two clear examples.

33 For methods of projection which associate multiple environmental features with the same picture property, the derived features in the map are disjunctive.
As this example makes clear, the type of projective feature map associated with a picture depends on the operative system of depiction. Variation in the *marking* condition of these systems determines the kinds of features that are associated with each point. While variation in the *projection* condition will determine the specific angles associated with each point. A parallel projection system, for example, will determine a different, uniform set of angles, as the following diagram illustrates.\textsuperscript{34}

These maps in turn reveal the structure of the pictorial space imposed by the Projection Principle on each picture. In the case of linear perspective, the pictorial space has a *center*, relative to which every object or property in the picture’s content is assigned a three-dimensional polar direction. This gives such pictures their “perspectival” quality. By contrast, for parallel projection, the pictorial space is “flat”, locating objects directly in front of the picture plane. Given a point $p$...
on an arbitrary picture, the positions in pictorial space which \( p \) may represent will vary according to system. Parallel projection of the kind illustrated here will locate the depicted point directly in front of \( p \)'s position on the picture plane; perspective projection, on the other hand, will locate the depicted point an a radial angle from \( p \), relative to the central viewpoint.

Thus the Projection Principle associates every picture with a projective feature map. Each such projective feature map in turn corresponds to that picture's projective content, the aspect of a picture’s content contributed by the Projection Principle. Such content represents precisely those features which are held in common by every scene which is projectively compatible with a given picture— that is, by every scene not ruled out by the Projection Principle. By construing such common features as a projective feature map, the Projection Principle may be understood not merely as a constraint on content, but as the source of the most fundamental layer of pictorial content.

5 Comparisons

5.1 Other Projection Theories

The Projection Principle grows from a family of proposals in the philosophical literature which also seek to link pictorial content with projection. While the Projection Principle is the only such account to be formulated within a general semantic framework, I'll assume that the other accounts canvassed here are roughly commensurable.

To begin, the Projection Principle imposes only a necessary condition on content or accuracy. Greenberg (2013, pp. 249-253), by contrast, defends an account according to which projection is both necessary and sufficient for accuracy. Though my discussion here owes much to Greenberg’s framing of the issue, in this respect my proposal diverges sharply from his. Greenberg’s bi-conditional is perhaps plausible when one focuses exclusively on projective content; but it has unpalatable consequences if one considers a picture’s overall content. Indeed, as Sober (1976,

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35I originally defined viewpoints as pairs of projection sources and picture planes. If these elements are are allowed to vary with respect to one another among scenes within the content of a picture, then the Projection Principle does not determine a unique projective feature map for every picture, but a set of such maps. However, I think there is good reason to think that these elements do not vary with respect to one another within the content of any one picture. I suspect this relationship is fixed by the operative system in some cases, and by the intentions of the artist in others.

36Projective content is analogous to what Kulvicki (2006, ch. 3) terms bare bones content. See the discussion of Kulvicki’s view in the next section and in the appendix for a detailed comparison.

37Literature on depiction in psychology focuses on the content of the visual perception of pictures, rather than the content of pictures themselves. See, for example, Rogers 1995; Ittelson 1996; Koenderink and Doorn 2003; Cutting 2003; Vishwanath, Girshick, and Banks 2005. Though these studies have not converged on a specific projection-based constraint, it is widely assumed that perceived spaces will be at least approximately constrained by the projective geometry of light at the eye.

38Howell (1974) is a partial exception; while his focus is a formal semantics for sentences about pictures, he does develop a formalized notion of pictorial space that makes contact with possible-world semantics. However, for Howell, pictorial spaces are defined as geometrical structures which are located at worlds, rather than as logical constructs made up of worlds. As a consequence, they do not directly determine accuracy conditions.
p. 113) observes, if projection is sufficient for accuracy, then a picture’s content is accurate at any scene the picture can be projected from, no matter how bizarre, as in Picture E discussed in Section 1. But then the overall content of a picture would simply be its projective content; it would include the angular location of low-level features, but neglect any of the high-level content we intuitively associate with pictures.\footnote{A more charitable reading of Greenberg (2013) is possible. Greenberg focuses on the accuracy conditions of pictures, rather than those of the contents of pictures. It’s possible that his formulation is correct for a distinct notion of picture-accuracy; but then this cannot be the notion of accuracy which figures in the accuracy-conditions specified by pictorial content. In that case, the theory, though plausible, has less significance for pictorial semantics generally.}

A different class of theories make projection only a necessary condition on pictorial content, so are more closely aligned with the Projection Principle in this respect. Perhaps the most direct antecedent of the Projection Principle is with the Occlusion Shape Principle proposed by Hyman (2006). The occlusion shape of an object is, roughly, the shape optically projected from an object to an intersecting plane along a given “light of sight.” The Principle is then formulated as follows: if a part P of a picture depicts an object O, then “the occlusion shape of O and the shape of P must be identical” (2006, p. 81). Assuming that the sum of objects a picture depicts always determine a specific set of scenes, and reading “depicts an object” as accurately depicts an object, the principle expresses a constraint very much like the Projection Principle for Pictorial Accuracy, at least with respect to overall shape.

But the Projection Principle expands upon the Occlusion Shape Principle in at least three important ways. First, it make the definition of viewpoint, and the role it plays in the constraint on content completely explicit.\footnote{This has consequences for their treatment of specific types of depiction. Since the Occlusion Shape Principle is modeled after optical projection to the eye, it doesn’t distinguish the elements of picture plane and projection source, which in the present framework make up a viewpoint. As a consequence, the principle fails for systems of anamorphism, which involve projections where the projection source and picture plane are positioned at acute angles with respect to one another. Since the Occlusion Shape Principle is invalid here, Hyman (2006, pp. 93-98) is compelled to introduce an additional principle for the special case of anamorphosis. By contrast, the Projection Principle handles anamorphosis as a yet another system of depiction, albeit one which imposes distinctive constraints on viewpoint.} Second, Hyman’s Principle only adverts to occlusion shape, and he posits independent principles for occlusion size (pp. 98-99) and for color (pp. 99-104). By contrast the Projection Principle integrates all of these features into a single well-defined rule.\footnote{Even for Hyman’s color and size principles, it will be impossible to state these conditions precisely without ultimately relying on the spatial organization provided by a projective transformation.} Finally and most significantly, Hyman’s Principle seems to be keyed to only a narrow range of systems of depiction. In general, he writes as if occlusion shape can always be defined relative to a point-sized projection source (p. 76), with projection lines that follow the behavior of light (p. 77).\footnote{He remarks, for example, that occlusion shape would be effected by refraction. (2006, p. 77)}

The Projection Principle expands this purview to include even those systems whose geometries deviate from both optics and linear perspective.

A closely related proposal comes from Kulvicki’s (2006) account of pictorial content. Kulvicki posits a basic layer of pictorial content, called bare bones content, which is in turn defined in terms of projection: “whatever scenes could have resulted in a particular [linear perspective picture] via

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a perspective projection count as parts of the [bare bones] content of the picture.” (Kulvicki 2006, p. 59) This formulation, in fact, expresses only the sufficiency of projection for bare bones content, but the necessity clause is clearly implied in addition. So we can think of the bare bones content of a picture as the set of all scenes which project to the picture. (This is roughly equivalent to the notion of projective content defined above. See the appendix for a more detailed comparison.) Bare bones content itself is understood to constrain a picture’s overall content, so that whatever the final interpretation of a picture, it must be compatible with the picture’s bare bones content. The result is a commitment very much like the Projection Principle for Content, since it makes projection a necessary condition on a scene to be part of a picture’s content.

The Projection Principle expands upon Kulvicki’s proposal again by making the role of viewpoint explicit and by clarifying relationship between projective geometry and the theory of line and color. The Projection Principle also extends to a wider set of representational systems than Kulvicki’s constraint, since he limits his to those systems which exhibit a formal property he terms “transparency.” A thorough discussion of this idea would take us too far afield. Suffice to note that a commitment to preserve transparency ultimately motivates Kulvicki to endorse a non-standard theory of pictorial structure, which has the effect, as he acknowledges, of weakening the logical strength of his proposed projection constraint. As a result, Kulvicki’s constraint turns out to be an entailment of the Projection Principle, but is significantly less semantically informative. These points are spelled out in the appendix.

A third view in the same vein is due to Howell (1974, p. 90), who defines a picture’s picture space as a type of spatial array for which the picture could be a projection. He also allows that different picture spaces may be associated with different forms of projection, so implicitly endorses a constraint much like the Projection Principle. However, because Howell’s goal is to provide a formal semantics for sentences which attribute content to pictures, rather than for pictures themselves, the semantic properties of pictures are never clearly set out. Furthermore, like other accounts discussed here, Howell’s analysis focuses almost exclusively on the spatial aspects of projection, neglecting the critical ingredients of line and color. Finally, his definition of picture space intermingles the projective assumption, which I endorse, with the contentious claim that every point in the picture space occupies a determinate depth relative to the viewpoint. This claim appears too strong, since many pictures represent objects as located at an indeterminate depth. The Projection Principle avoid this commitment.

A last group of theories emerges from the works of Peacocke (1987), Budd (1996), and Hopkins (1998); these authors enlist the concept of projection in a very different way, to elucidate the classical resemblance theory of projection. I discuss these views in detail in the next section.

In sum, while the idea that pictorial content is constrained in some way by projection has been

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43To be precise, Kulvicki holds that for a system to be pictorial, it must exhibit transparency, or near enough. (See ch. 3, especially p. 76.) I discuss my disagreements with this claim in Section 2, in particular footnote 26.
explored by a number of scholars, the Projection Principle amplifies this line of thought through its attention to viewpoint, systems of depiction, the treatment of line and color, and its articulation within a systematic semantic framework.

## 5.2 Similarity Theories

The class of “resemblance theories” of depiction has taken many forms over the years. At its broadest, it includes any theory that posits a systematic structural correspondence between a picture and its content—and contrasts with theories which ground depiction primarily in arbitrary conventions or in human perceptual response. Under this conception, projection-based theories are a species of resemblance theory.\(^4\)

Still, there are important distinctions to be drawn among different ways of characterizing structural correspondence. The projection-based approach should be contrasted with the popular claim that depiction is grounded in relations of restricted similarity (Abell 2009; Blumson 2009a; Greenberg 2013). I call views which follow this doctrine, SIMILARITY THEORIES. Similarity theories hold that the content expressed by a picture is both limited by, and in large part explained by, similarities between the picture and the content. The traditional expression of this idea construes similarity as “resemblance” (now in a narrower sense): between the experience or optics of looking at a picture, on one hand, and that of looking at the space it represents, on the other. In this approach, pictures must resemble the spaces they express. This strategy is taken up by Peacocke (1987), Budd (1996), and Hopkins (1998), and discussed in detail below.

An alternative idea construes similarity as spatial isomorphism, understood as a kind of abstract structural “match” between two spatially defined systems (e.g. French 2003). On this view, pictures must be isomorphic to the spaces they express. Depiction is “direct” here because, by coming to know the structure of a picture, one directly comes to know (one aspect of) the structure of its content. Yet despite its apparent abstraction, isomorphism is just another form of similarity—similarity with respect to abstract, relational features—so counts as a similarity theory as well.

Projection-based theories and similarity theories express deeply divergent ideas about depiction, dividing over formulation, extension, and ultimately, motivation. By conceiving of pictures as re-presentations or structural copies of their contents, similarity theories require primarily that a picture and its spatial content be similar, defined as sharing a certain set of properties. But as Greenberg (2013, pp. 279, 283) has emphasized, projection is a transformation, one that can only be defined in terms of both the similarities it preserves and the deformations it introduces. To be sure, projection does entail certain PROJECTIVE INVARIANTS—properties of a scene that always survive projection to the picture plane—but projection goes beyond invariants, requiring systematic difference as well. As a consequence, whereas similarity relations are always symmetric and

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\(^4\)Hyman (2006, p. 111) describes his projection theory as “the defensible residue of the resemblance theory of depiction.”
reflexive, projections are not. For these reasons, the projection analysis cannot be reduced to one of resemblance or isomorphism.45

These differences in conception lead to differences in extension. Greenberg (2013) argues that similarity theories in general are extensionally inadequate when it comes to the full variety of systems of depiction. In the same vein, I’ll argue here that one of the most promising versions of the resemblance approach to emerge from the philosophical literature fails to live up to the standard of the Projection Principle. Developed by Peacocke (1987), Budd (1996), and Hopkins (1998) this cluster of views is of special note in the context of this essay, for each precisifies the relevant notion of resemblance using the concept of projection.

In different ways, each of these authors pursue the common thought that, for a picture to (accurately) depict a scene, the picture and the scene must look alike in relevant respects. Further, how something looks, at least with respect to shape, is defined in terms of its VISUAL FIELD SHAPE, understood as one the apparent proximal shapes that make up the phenomenal visual field; it is further assumed that an object’s visual field shape is roughly an optical projection of the scene before the viewer.46 Putting these ideas together, the claim is that, for a picture to be accurate at a scene, the visual field shape projected by the picture, and the visual field shape projected by the scene, must be similar with respect to shape. I’ll call this the Resemblance Principle.

To bring out the concrete differences between Projection Principle and the Resemblance Principle, I’ll take the liberty of translating the resemblance theorist’s phenomenological vocabulary into the geometrical vocabulary of this essay. Here I’ll let V be a linear perspective, optical method of projection, as near as possible to that encoded by the human visual system; and I’ll call projection by V, VISUAL PROJECTION. The idea is that the geometric shapes which are the result of visual projection will stand in for visual field shapes. Now, letting \( \text{proj}_V(\cdot) \) be V’s associated projection function, we can formalize the Resemblance Principle along side the Projection Principle within the framework of scene-based semantics:

\[
(1) \text{ if } [P]_{S,c} \text{ is accurate at } (w, v), \text{ then } \text{proj}_S(w, v) = P
\]

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45 The distinction between similarity- and projection-based theories is sometimes clouded because projective invariants are superficially analogous to the properties held fixed in a similarity theory. But not only are projective invariants insufficient to define projection, as noted in the text, projective invariants do not even ground substantive similarities between picture and scene. For example, the property of line straightness is invariant under linear perspective projection: if a line is straight in the scene, its projection will also be straight. Yet the reverse is not true. For there can be straight lines in the picture which are not projected from straight lines in the scene—for example, curved lines in the scene which lie along planes that intersect the viewpoint and are perpendicular to the picture plane. Thus while line straightness is an invariant under perspective projection, it is not in general a property shared by a picture and its content. Thus projective invariants are an impoverished starting point for similarity theories.

46 This idea is expressed differently by the different authors. Peacocke (1987) writes of shapes of regions in the visual field (p. 385). Budd (1996) uses the idea of two-dimensional aspects of the visual field (pp.158-159), understood as a 2-dimensional abstraction of 3-dimensional perceptual space. And Hopkins (1998) speaks in terms of the outline shape of an object, relative to a viewpoint. He defines outline shape as the apparent angle subtended by the object relative to a viewpoint; this is not quite the same as visual field shape, but the difference is harmless, since every visual field shape determines a unique outline shape, and vice versa.
if \([P]_{S,c}\) is accurate at \(\langle w, v \rangle\), then \(\text{proj}_V(w, v) \approx \text{proj}_V(P, v^*)\)  
(Resemblance Principle)

where \(v^*\) is a canonical viewpoint on \(P\).

Here \(\approx\) indicates a designated degree of similarity between the two projections, and \(v^*\) is a standard, frontally oriented viewpoint. A further simplification makes the two views easier to compare. Linear optical projection from an appropriate canonical viewpoint typically preserves all spatial features of a 2-dimensional surface except scale. Thus we may assume that, in general, \(\text{proj}_V(P, v^*) = P\), setting aside matters of scale. As a consequence, the Resemblance Principle normally entails:

\[
\text{(3) if } [P]_{S,c} \text{ is accurate at } \langle w, v \rangle, \text{ then } \text{proj}_V(w, v) \approx P
\]

which directly mirrors the Projection Principle; here the only differences are (i) the fixed role of \(V\)-projection, where the Projection Principle has a variable for any system \(S\), and (ii) the relation \(\approx\), which the Projection Principle replaces with identity.

In fact, the central challenge for the Resemblance Principle is precisely that it gives \(V\)-projection a fixed role, without allowing for meaningful variation by system of depiction. If \(\approx\) is understood to imply identity (or near identity) in surface geometry, then the theory falters immediately for any system of depiction that departs from \(V\). Systems of parallel projection, for example, produce accurate pictures which look noticeably dissimilar from their target scenes with respect to shape. In terms of geometry: a (\(V\)-projection of a) parallel projection of a scene and a \(V\)-projection of a scene differ significantly in surface shape.\(^{47}\) Since the Resemblance Principle requires similarity of shape for accuracy, many accurate pictures in systems of parallel projection (and other deviations from \(V\)) will wrongly be deemed inaccurate. In terms of content, the Resemblance Principle implies that non-perspective projection be interpreted as perspective projections, leading to the unpalatable conclusion that parallel projections, for example, almost always express spatially distorted content.\(^{48}\)

On the other hand, suppose \(\approx\) is understood sufficiently loosely that it is not counter-examined by systems of parallel projection. Instead of identity of surface geometry, it is defined as loose similarity of surface geometry. (The notion of similarity may be entirely relational, as with

\[^{47}\text{That is: where } L \text{ is the system of parallel projection, then for many scenes } \langle w, v \rangle: \text{proj}_V(\text{proj}_L(w, v), v^*) \neq \text{proj}_V(\text{proj}_L(w, v), v^*) \text{ because } \text{proj}_V(\langle w, v \rangle) \neq \text{proj}_L(\langle w, v \rangle). \text{(Here I gloss over the difference in viewpoint structure between the two systems.)}\]

\[^{48}\text{If the defect of the Resemblance Principle is its insensitivity to variation in systems of depiction, it might be thought that the theory could be rescued by relativizing its notion of projection to a system. Such an account might have the following form: }\]

\[
\text{if } [P]_{S,c} \text{ is accurate at } \langle w, v \rangle, \text{ then } \text{proj}_S(w, v) \approx \text{proj}_S(P, v^*)
\]

But Greenberg (2013) argues that such theories also run afoul of variation among systems of depiction. The problem is that, for many systems, most pictures are such that \(P \neq \text{proj}_P(P, v^*)\) — as Greenberg (2013, pp. 263-271) notes, these include systems of curvilinear perspective, systems of scale, and a variety of color-shifting systems like technicolor or brown-tinting. But this fact, together with the proposed principle, has the bizarre consequence that, for many scenes, if a picture is a projection of that scene it cannot be accurate. For if \(\text{proj}_S(w, v) = P \text{ and } P \neq \text{proj}_S(P, v^*)\), then \(\text{proj}_S(w, v) \neq \text{proj}_S(P, v^*)\). Then by the proposed constraint, \([P]_{S,c}\) could not be accurate at \(\langle w, v \rangle\).

§5 Comparisons
Budd’s (1996, pp. 164-165) concept of “structural isomorphism.” Here the definition of similarity must be lax enough to allow that not only perspective projections, but also parallel projections may be accurate. Thus it must cover not only the accurate perspective projection L, of two parallel tracks extending in space away from the viewpoint, but also the accurate parallel projection M of the same scene. A definition of similarity which satisfied these demands would have to be extremely abstract and afford considerable spatial latitude.

At this point, however, it becomes clear that the Projection Principle is considerably more determinate than the Resemblance Principle. The Projection Principle puts specific quantitative requirements on the distribution of surface features in pictorial space, whereas the proposed Resemblance Principle would only be able to impose highly approximate constraints. As Budd (1996, p. 164) himself acknowledges, it captures “very little of the detail of [pictorial] representations.”

These lessons seem to generalize. Whereas the Projection Principle offers a determinate and detailed semantic constraint, similarity theories in general appear to offer only open-ended, indeterminate requirements. By limiting themselves to shared properties, similarity theories foreclose the possibility of capturing the rich structural relationships that emerge between a picture and the space it expresses.

5.3 Perceptual Theories

I turn finally to compare the Projection Principle with the family of perceptual theories which ground depiction directly in visual perception, such as those of Schier (1986), Wollheim (1987), Lopes (1996), or Newall (2011). Schematically, views of this kind seek to derive the content associated with a picture from the content that the visual system would generate when exposed to

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49To get more traction, the resemblance theorist might try to relativize similarity to system of depiction. But it is hard to see what L and the scene have in common which M would not. But then the notion of similarity for perspective projection would fail to rule out M as accurate relative to the perspective system.

50There are other kinds of accounts which might also be called “similarity theories”, but which cast the concept of similarity in quite different roles. One “meta-semantic” theory holds that for a system to be pictorial (or perhaps, iconic), similar pictures must express similar contents— hence the space of pictures is similar or isomorphic to the space of contents. (Thanks to John Carriero and Matthew Fulkerson for suggesting this idea.) The Projection Principle is, as far as I can tell, fully compatible with this proposal.
the picture under suitable circumstances (e.g. when looking at the picture in normal lighting). For example, Wollheim (1987) holds that for a picture to express a given space as its content, it must be possible for a suitable viewer to see that space in the picture—where the critical notion of “seeing in” is defined at least in part by the normal operation of visual perception. Other authors, like Schier (1986) and Lopes (1996), hold that for a picture to attribute a property as part of its content, as suitable viewer must visually recognize that property in the picture.

In Sections 1 and 2 I defined systems of depiction as those which bear certain affinities to human vision. In this respect, the present account is aligned with perceptual theories of depiction. Nevertheless, the projection-based analysis and these perceptual theories cast visual perception in different theoretical roles. For traditional perceptual accounts, the role of perception is semantic—it helps to explain why individual pictures have they content they do, in virtue of the perceptual contents possessed by an idealized viewer. By contrast, I hold that the role of perception is meta-semantic—it helps explain why some rules and not others are selected as the interpretive principles of depiction. But at least for the low-level phenomena considered here, I do not draw any direct connection between the contents of pictures and the contents of perceptual states.

The degree of contrast between the Projection Principle and perceptual theories of depiction depends on which aspects of pictorial content these accounts aspire to explain. If their interest is limited to explaining why pictures depict certain individuals as representing certain high-level properties, as appears to be the case of Schier and Lopes, then there may be no conflict with the Projection Principle. The two are simply aimed at different domains. But if perceptual theories aim to explain the spatial geometry of pictorial content, as Wollheim’s (1987) remarks suggest, then they are in tension with the Projection Principle.

The obstacle for perceptual accounts, conceived as accounts of pictorial space, is once again the diversity of systems of depiction. Especially in its early stages, human visual perception is thought to work in a manner that is relatively fixed, both among human perceivers, and within perceivers across contexts. The influence of world knowledge and reasoning on low-level spatial vision is negligible (Pylyshyn 1999). It is not clear how a set of fixed visual process could yield the interpretive variability introduced by the many systems of depiction. The challenges are now familiar: early vision normally treats certain kinds of converging lines in the retinal image as indicative of parallel edges in the environment. But in parallel projection, for example, converging lines on the page can only indicate converging edges in the scene. Applying normal visual perception to such cases yields incorrect interpretations.51

Another area of difficulty for perceptual accounts stems from cases of optical illusion. Suppose that a friend puts a reproduction of the Müller-Lyer illusion on her wall, and I make a drawing of

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51Perceptual theorists may wish to speculate that the visual system is in fact flexible enough to incorporate non-optical systems of depiction in its production of perceptual representations. If true, this would make the two accounts compatible; but it would then be relatively uninformative to say that pictorial content is determined by the exercise of vision. One would then want to know what kind of vision, leading one back to an analysis in terms of specific methods of projection.
it. Suppose my drawing is a perfect linear-perspective projection of the scene before me, and it is drawn from a viewpoint such that the two equal-length bars of the illusion are depicted by equal-length lines on the picture plane. Perceptual theories seem to imply that by necessity my picture cannot be accurate: because of the illusion, even in ideal conditions normal viewers will always misperceive the difference in length between the two lines on the picture plane, and because of this misperception, they will attribute a difference in length to the two represented bars in the content. Yet, intuitively, it should be possible for a perspective projection of a scene to be an accurate depiction of that scene. The Projection Principle assumes no constitutive semantic link to human vision, so makes no such prediction. Such considerations are not decisive, but they suggest another class of cases which is easily reconciled with the projection analysis, but remains a challenge for perceptual theories.

In sum, theories which aim to characterize pictorial space directly in terms of normal human vision are unlikely to work out, for no simple equation links the spatial content of a picture to the content elicited by early vision when looking at a picture. By contrast, I’ve argued that Projection Principle is derived from the mechanisms of human vision, but cannot be identified with them. The Projection Principle captures the fact that depiction conforms with the general structure of vision, but allows that depiction also departs from vision in myriad ways.

6 Impurely Projective Systems

Thus far my discussion has focussed on a select class of systems of depiction which I shall call purely projective. Such systems are defined exclusively in terms of low-level spatial and chromatic properties, and they produce pictures which have a characteristically “mechanical” or “optical” look and feel. They include most forms of photography as well many of the drawing systems used in medical, engineering, architectural, and scientific contexts. But such systems do not exhaust the phenomenon of depiction, and in this section I’ll contrast purely projective systems with the remaining impurely projective systems, including systems of stylization, caricature, and indeterminate geometry. I’ll argue that Projection Principle does not immediately apply to these cases, but they nevertheless support the more general claim that all forms of depiction are grounded in projection, in one way or another.

I begin by with an account ofpurely projective systems: these are systems whose characteristic methods of projection can be defined as functions of only the local spatio-chromatic features of the scene projected.\footnote{Optionally, one might require that purely projective systems are uniform: they use a single kind of spatial relation to determine the structure of the entire picture plane. This excludes methods of projection which enlist different kinds of spatial structure for different regions of the picture plane. But nearly any violation of uniformity also violates locality.} Such methods depend only on local features of the scene in the sense that they can be defined entirely in terms of dependencies between very small regions of the scene and features of the picture plane. This means that the composition of the projected image can be
worked out point by point (or small region by region) in relation to the scene. These projections depend only on spatio-chromatic features in the sense that they are sensitive only to low-level spatial and chromatic features of the target scene, like the presence of edges, surfaces, local textures, and reflectance properties. For example, linear perspective line drawing is purely projective because it can be defined as mapping the edge-features of point-sized regions in the scene to color-features of point-sized regions in the picture. Discussions of the projection-based approach to depiction in the philosophical literature have focussed almost exclusively on purely projective systems like this.

By contrast, impurely projective systems are those systems of depiction in which features of the picture plane cannot be derived from local spatio-chromatic properties of the scene alone. Instead, they are typically sensitive to high-level properties of the scene, such as whether it contains people, animals, or plants, or particular individuals. In such a system, for example, how something is represented on the picture plane might depend on whether it is a person, a plant, or an article of clothing. It turns out that there are many different kinds of impurely projective system. They include, by my count, systems of stylization, systems of caricature, systems which involve more than one projective method, and systems which tolerate highly indeterminate geometry. Though each of these cases is governed by its own set of rules, and deserves dedicated discussion, I’ll focus here on stylization as representative of the broader class of impurely projective systems.

Stylization arises when artists follow general prescriptions for drawing different types of objects. Following Gombrich (1960, ch 5), I call such prescriptions schemas. Schemas dictate different “ways” of drawing people, cats, trees, and so on. Such schemas are precisely what children learn when they learn, for example, how to draw a house, a cat, or a tree. Rather than making their drawings conform principally to the way a given object is in fact shaped—as with pure projection—artists employing stylization instead make their drawing conform with an antecedently established schema. Far from a specialized case, stylization dominates art history. Virtually all pictures created before the Italian Renaissance contain some degree of stylization, and a large share of picture-production since continues this tradition. Here are two forms of stylization, one from a contemporary crossing sign, the other from a tomb in the Theban Necropolis, dating from the 15th century BCE.

§6 Impurely Projective Systems
Whatever the precise spatial content of Picture N, it is clear that it depicts a normally-shaped person walking on a path or crosswalk. The fact that the head of the stick figure is drawn with a large black circle does not imply that, in the content, there is a large sphere floating above a stick-like form. Rather, this apparent violation of projection is the result of the application of a schema for drawing humans. Similarly, Picture O depicts a group of men in a funeral procession. The fact that, in the picture, we see the frontal plane of their chests, but the sides of their feet, does not imply that their bodies are impossibly twisted around (Hagen 1986, pp. 169-170). Instead, they are again produced from an antecedent prescription for drawing people, in manner to some degree independent of their actual shape. In general, to the extent that stylized images employ such schemas, they depart from pure projection.

Stylized systems are not purely projective; therefore they do not conform with the Projection Principle, so long as projection is understood to be pure projection. A more complex question is whether these systems would conform with the Projection Principle were we to lift the restriction that the projection in question be pure. To do so, it would have to be possible to define a projection function that mapped scenes to fully stylized images. Whether this idea can be carried out is an open question and a technical one; I leave such puzzles about the semantics of stylization to further inquiry.

Though stylization is not purely projective, I wish to argue that stylized systems of depiction are still grounded in projection, albeit in a looser and less direct sense than purely projective systems. This grounding in projection emerges in two ways. First, the schemas adopted in stylized depiction are not, in general, entirely arbitrary. Instead, they are loosely derived from projections from stereotyped viewpoints. The bubble-figures in crosswalk signs illustrate this principle; while they are clearly not pure projections, the overall organization of legs, arms, torso, and head correspond loosely to the positions those objects would have in a pure projection from a side view. (A projection from above or below would have a rather different layout.)
tem, the schema for a type of object X is derived from the pure projection(s) of a standard instance of X from some canonical viewpoint (or viewpoints).^{55}

The second sense in which stylized pictures are grounded in projection arises at the level of global spatial organization. Even when stylized systems use schemas to determine the representation of individual objects, they still use a projective organization of space to determine the relationships between these objects. Thus the positions of stylized figures on the picture plane tend to correspond roughly to the positions that would be purely projected from the pictorial space. At the most basic level, the use of up-down and left-right dimensions on the picture plane to indicate the angular organization of pictorial space relative to a viewpoint, is a hallmark of projective structure. Here it is notable that both of the stylized images above are distinctly perspectival. Picture O, for example, expresses a space in which the viewpoint is positioned to the side of the procession, and the depth relations between the figures are indicated by occlusion; further, the various figures (as well as the limbs of those figures) are each positioned at different, quantitatively significant angles relative to one another. And Picture N expresses a pictorial space in which the viewpoint is situated approximately level with the head of the figure crossing the road, and the depth of the walkway edges away from the viewpoint is reflected in their differential position on the picture plane. But these differences only gain semantic significance through a projective interpretation. Thus, even in stylization, projection from a viewpoint plays a central, organizing role.^{56}

This pattern of facts—local violation but global conformity with pure projection—is endemic not only to stylization, but also caricature, pictures which employ multiple methods of projection, and those with distorted or inconsistent geometry. It is in this sense that such systems are not purely projective, but impurely projective. While impurely projective systems do not conform to the Projection Principle, narrowly construed, they are nevertheless grounded in depiction in a more general sense.

7 Conclusion

The subject of this essay is the Projection Principle. I’ve argued that a core class of purely projective systems conform to it. What’s more, even impurely projective systems are based on the global geometry of projection, despite local deviations. In either case, the basic viewpoint-centric structure of pictorial content arises from the scaffolding of projection. Ultimately, an analysis of pictorial representation based on projection has much to recommend it. It captures basic facts of extensional adequacy. It characterizes the nature of the correspondence between a picture and the

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^{56}As Hagen (1986, pp. 168-176) has argued, the global space of ancient Egyptian painting is usually determined by some form of parallel projection. A clear sign of this is that objects do not become smaller as they recede into space. In general, however, stylized system may employ any type of global projection, including perspective projection. Willats’ and Hagen’s discussions of pictorial systems are replete with examples.
space it expresses while making room for significant variation among systems. And it implies a precise account of the viewpoint-centric structure of these spaces. Finally, it elucidates the overlap between visual perception and pictorial interpretation, but it is able to respect the independence from vision which so many systems of depiction require. In sum, pictorial representation is, at its core, representation grounded in projection.
Appendix: Bare Bones Content and the Projection Principle

In this appendix I argue that, in the context of Kulvicki’s (Kulvicki 2006, ch. 3) claims about pictorial structure, the constraint he offers on pictorial content is significantly weaker than the Projection Principle. To begin, Kulvicki makes use of the notion of bare bones content, understood roughly as the set of scenes which could project to a picture, relative to a system. Letting $B_S(\phi)$ be the bare bones content of $\phi$, relative to $S$, we can render Kulvicki’s definition within a scene-based semantics, as follows:

(4) $B_S(P) = \{ \langle w, v \rangle \mid \text{proj}_S(w, v) = P \}$

Next, Kulvicki holds that bare bones content constrains a picture’s overall content. That is:

(5) $[P]_{S,c} \subseteq B_S(P)$

Put together, these entail:

(6) $[P]_{S,c} \subseteq \{ \langle w, v \rangle \mid \text{proj}_S(w, v) = P \}$

which is equivalent to the Projection Principle for content:

(7) if $\langle w, v \rangle \in [P]_{S,c}$ then $\text{proj}_S(w, v) = P$

However, motivated to preserve a property of systems he calls “transparency,” Kulvicki takes a non-standard view of pictorial structure. Rather than defining pictorial structure metrically, he holds that pictorial structure consists only of projective invariants. Since pictorial structure is the only feature of a picture which determines bare bones content, it follows that all pictures which share projective invariants have the same bare bones content (Kulvicki 2006, p. 57).

To compare the resulting idea with the Projection Principle, I’ll translate Kulvicki’s proposal back into the framework of this essay in which pictures are treated metrically. Kulvicki’s new bare bones content can be defined as the set of all scenes which project to something which shares projective invariants with the target picture. Since projections of a picture always share projective invariants with it, I can restate the definition of bare bones content, now $B^*$ as follows. (Here I’ll have to allow that the projection function can take pictures themselves as arguments—interpreted as worlds which contain those pictures.)

(8) $B^*_S(P) = \{ \langle w, v \rangle \mid \exists v' : \text{proj}_S(w, v) = \text{proj}_S(P, v') \}$

The resulting constraint on content can now be set side by side the Projection Principle:

(9) if $\langle w, v \rangle \in [P]_{S,c}$ then $\exists v' : \text{proj}_S(w, v) = \text{proj}_S(P, v')$ (Kulvicki’s Constraint)
(10) if $\langle w, v \rangle \in [P]_{S,c}$ then $\text{proj}_S(w, v) = P$ (Projection Principle)

The difference here cuts deep. Consider the diagram below in which pictures B and C are projected from A, thus share projective invariants with it:

§Appendix: Bare Bones Content and the Projection Principle
Intuitively, A, B, and C have different spatial content. The Projection Principle delivers this result: since each image can only be projected from scenes which the others could not, they cannot have the same content. (To be clear, any world which includes A will be is compatible with all three pictures’ contents. But no scene will be compatible with all three.)

But Kulvicki’s Constraint cannot capture this claim. Let L be the system of perspective projection. Perspective projection from a plane is reversible, modulo issues of scale; so if there is some v such that proj_L(P, v) = P', then there is some v' such that proj_L(P', v') = P. As a consequence, Kulvicki’s Constraint puts the same extensional requirement on each of A, B, and C. This is to be expected, given the background theory: if all three have the same pictorial structure, and structure determines bare bones content, then Kulvicki’s Constraint can’t distinguish between them. Thus the Projection Principle issues in a more determinate constraint that Kulvicki’s Constraint.

While the semantic differences between A, B, and C are in principle compatible with Kulvicki’s view at the level of what he calls “fleshed-out content,” only the Projection Principle actually delivers the relevant result. Since neither Kulvicki nor I have offered a detailed account of how fleshed-out content is determined, the Projection Principle goes farther, given the theoretical resources available, towards explaining these differences in a systematic manner.
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References


